This paper describes a scheme that uses holographic optics for realization of schlieren diffraction interferometer. The interferometer has been realized with single holographic optical element in conjunction with a collimated beam and the focal plane-modifying element. Multiplexing ability of holographic optics also enables simultaneous operation of more than one technique such as schlieren, shadowgraph and interferometric techniques. Combined schemes where one or more techniques could be applied simultaneously are useful for detailed investigations of phenomena like flow of compressible fluids, because each method has its own characteristic strengths and weaknesses. Procedures for the generation of holographic optical element and realization of various schemes have been presented.

Schlieren techniques are currently used for a wide range of applications, including test studies on phase objects, depicting deviations in light beams induced by density, temperature or refractive index gradients in combustion research such as laminar and turbulent flame fronts, shock and detonation waves, plasma diagnostics and other steep refractive index gradients associated with heat and mass transfer, or pressure changes, but not confined to these\textsuperscript{1\textendash}6. Need of a single beam for operation makes these techniques inherently stable, relatively insensitive to external vibrations and requiring fewer optical elements in contrast with conventional interferometry. One of the limitations to carrying out test studies on large size phase objects in almost all of the interferometric schemes is the difficulty of fabrication and high cost of large size, good quality optics. Recently, use of Holographic Optical Elements (HOEs) as an alternative to conventional optics has been successfully demonstrated for realization of various interferometric schemes\textsuperscript{7\textendash}15. Use of HOEs instead of conventional optics can drastically reduce bulkiness and high cost factors, which become more advantageous for space applications. Because of several attractive features offered by HOEs, such as light weight, compactness, ease of fabrication, multiple optical functions in a single element and their high functionality compared with conventional bulky optics, their use provides advantage in the construction of compact and low-cost optical systems. This paper describes a scheme for realization of schlieren diffraction interferometer\textsuperscript{1,16,17} using holographic optics to carry out test studies on phase objects in real-time. The system can operate independently as a schlieren diffraction interferometer or may be combined with other techniques for obtaining more precise information about the test field. The optical arrangement of the proposed set-up involves a simple alignment procedure and the interferometer works well with a single collimated beam.

**Principle of the method**

The method reported in this paper for realizing holographic optics-based schlieren diffraction interferometer involves the formation of a HOE. As shown in Figure 1, a converging beam $O$ is used in conjunction with a normally incident collimated beam $R$ to record the HOE on a holographic recording plate $H$. Illumination of processed HOE with reference beam $R$ reconstructs the beam $O$. A viewing diaphragm (knife-edge, mirror-edge or folding mirror) at the schlieren focus of this reconstructed beam converts the phase variations of the test object placed in the test beam, which in our case is the collimated beam $R$ itself, into an intensity pattern. The complex amplitude distribution of the object beam and the reference beam can be considered as:

$$O = (O_r/r_1) \exp(-jkn_o \cdot r_1),$$

$$R = O \exp(jk n_o \cdot r),$$

where $n_o$ and $n_r$ are unit vectors along the direction of propagation of the object beam and reference beam respectively; $r$ and $r_1$ are displacement vectors; $k = 2\pi/\lambda$, where $\lambda$ is wavelength of the light used and $j = \sqrt{-1}$. $O_r$ and $O$ are the amplitude distributions of corresponding beams. The amplitude transmittance of the processed $H$ is given by\textsuperscript{18}:

$$t_1 \sim |O + R|^2.$$  \hspace{1cm} (2)

The complex amplitude of the transmitted field from $H$, upon illumination with reference beam $R$, is

$$U_1 = R_1 - R |O_r|^2 + R |O|^2 + O |R|^2 + O^* R^2.$$  \hspace{1cm} (3)

**Figure 1.** Schematic recording set-up for the formation of holographic optical element for schlieren diffraction interferometry.
If a phase object $S = \exp(j\phi)$ is inserted in the collimated beam $R$, the beam will be phase modulated accordingly and this phase modulated beam $R' = O \times \exp\{jk_n_s \cdot r + \phi\}$ will illuminate the processed $H$ instead of beam $R$, giving

$$U'_1 = R'_1 \sim R(O)^2 + R|R|^2 + OR'R^* + O^*R'R. \tag{4}$$

We can consider $|R|^2$ to be constant across $H$, as a plane reference beam $R$ is used for illumination of $H$. Thus, only the third term on the right-hand side of eq. (4) is of interest to us as it represents reconstructed beam $O$ modified with the phase object $S$, i.e.

$$OR'R^* = O_o \exp\{jk_n_s \cdot r + \phi\}|R|^2,$$

$$U'_1 \sim (\text{Constant}) O_o \exp\{jk_n_s \cdot r + \phi\}. \tag{5}$$

It becomes obvious from eq. (5) that the phase object introduced in the collimated illuminating beam $R$, modulates the reconstructed beam $O$. This reconstructed field is further manipulated with different schlieren elements to convert its phase variations into an intensity variation to carry out optical test studies.

**Experimental details**

In our experiments, a continuous-wave helium–neon laser (Coherent Inc., USA 35 mW) has been employed as the coherent light source for recording the HOE and for the realization of the schlieren diffraction interferometer. The experimental arrangement for the formation of HOE is schematically shown in Figure 1. A collimated reference beam $R$ was generated using a collimating lens $L_1$ (50 mm diameter; $f/3$) in conjunction with the beam expander assembly $B_2$. The convergent beam $O$ was created using a good quality telescopic set-up of two lenses $L_2$ and $L_3$ (100 mm diameter; $f/4$) in conjunction with the beam expander assembly $B_2$ to generate aberration-free focus $F$. The shear plate interferometric technique was applied to ensure the optical quality of the collimated beams and Ronchi test technique was used for optical correction of the converging beam for astigmatism and coma, which would otherwise be introduced by the off-axis arrangement. Agfa-Gevaert BE75HD plates have been used for hologram recording and standard Kodak D-19 developer and R-9 bleach bath solutions have been used for chemical processing. The positioning of focal plane schlieren element, i.e. knife/mirror-edge or folding mirror (20 mm $\times$ 40 mm $\times$ 2 mm, SiO$_2$-protected, front surface silver-coated, reflectivity ~94%) as shown in Figure 2, provides the test results of the respective geometries. The results presented here have been captured frame-by-frame with a Canon S-50 Power Shot digital camera (1024 $\times$ 768 pixels) in white-balance settings.

**Holographic optics-based schlieren diffraction interferometer**

A schlieren diffraction interferometer has been created using a knife-edge at the reconstructed focus, as schematically shown in Figure 2. As an instrument, a schlieren apparatus is sensitive to transverse refractive index gradients in the test section. Conventionally, schlieren phase visualization device detects the slopes of index gradients where relative magnitude of gradients is estimated by observing the shadow patterns. The first area to become dark will have the positive gradients, followed by the flat areas, and finally the negative gradients. Recently, it has been shown that the conventional knife-edge diffraction pattern could be broadened in such a manner that a single diffraction fringe covers the field of view. This condition arises when two point sources (the focus and the secondary source situated on the knife-edge) coincide with each other, i.e. when the knife-edge diffracts light from the Airy disk. The broadening of the diffraction fringes has two main advantages over the conventionally used method of blocking the Airy disk. First, it enhances the sensitivity of the schlieren instrument by effectively using diffraction at the schlieren element, which serves as a limiting factor in the conventional schlieren systems. Second, in the situation where single diffraction fringe covers the field of view, schlieren element diffracts light from the Airy disk (containing about 84% of the incident light), resulting in a stronger diffraction light as the reference beam and thereby high contrast in the schlieren interferogram. According to the Maggi-Rubinowicz boundary diffraction wave theory, the diffracted field at the observation plane is given by

$$U(P_i) = U^R(P_i) + U^\ell(P_i), \tag{6}$$

where

$$U^R(P_i) = \begin{cases} \exp\{jR\} & \text{when } P_i \text{ is in the direct beam}, \\ 0 & \text{when } P_i \text{ is in geometrical shadow}, \end{cases} \tag{7}$$

and

$$U^\ell(P) = (1/4\pi)\int \exp\{jk(r + s)\} \cos(\mathbf{n}\cdot \mathbf{s}) \sin(\mathbf{d}\cdot \mathbf{s}) \, ds. \tag{8}$$

where $R$ is the distance from the source to the point of observation $P_i$, $r$ is the distance from the source to the knife-edge $K$, $s$ the distance between a typical point on the knife-edge and $P_i$, $\Sigma$ the boundary of the illuminated part of $K$, $dl$ the infinitesimal element situated on $\Sigma$, $\mathbf{n}$ the unit vector outward normal to the plane of the knife-edge, $\mathbf{s}$ the unit vector from observation point $P$ towards the integrated point on $\Sigma$; $\mathbf{d}$ the unit vector along the direction of infinitesimal element $dl$ and $j = \sqrt{-1}$. Here $U^R$ propagates according to the laws of geometrical optics and is known as the geometrical wave, while $U^\ell$ is generated from every point of

![Figure 2. Schematic experimental set-up for realization of holographic optics-based schlieren diffraction interferometer.](image-url)
the illuminated boundary of the knife-edge and is called the boundary diffraction wave. The contrast of schlieren interferogram could further be enhanced by replacing the conventionally used knife-edge with the mirror-edge\textsuperscript{16}, to get finer details of the test object. Thus, schlieren techniques could be used to retrieve information about gradient slope as well as interferometric results with proper adjustment of the knife-edge in the focal plane. However, the line integral in eq. (8) shows that the schlieren diffraction interferometer still provides information about transverse phase variations only. Figure 3 shows a typical infinite mode knife-edge diffraction pattern where a single diffraction fringe fills the field of view, obtained using holographic optics. Figure 4\textit{a} shows the typical test results on an optical glass plate in HOE-based schlieren diffraction interferometer. Test results on the same glass plate with conventional optics are shown in Figure 4\textit{b}. Here during recording the results with conventional optics, an iris was used to reduce the collimated beam width from an initial 100 mm diameter to a 50 mm diameter to obtain a similarity between the results. These results show that the schlieren diffraction interferometer provides the same information with holographic optics and conventional optics.

The same set-up could also be used for realization of beam-folding interferometer, where the diffracting element has been replaced with a folding mirror\textsuperscript{20,21}. Here the folding mirror superimposes two halves of the incident wavefront to obtain test results of phase objects interposed in the collimated beam. The set-up of the beam-folding interferometer using holographic optics has been schematically shown in Figure 5. A typical interferogram obtained with HOE-based beam-folding interferometer is shown in Figure 6. Figure 7\textit{a} shows the typical test results on an optical glass plate in HOE-based beam-folding interferometer. Test results on the same glass plate with conventional optics are shown in Figure 7\textit{b}. Here also the diameter of conventional optics was reduced using the iris as described previously. Comparison of these results depicts that the results using HOE optics give almost the same information, which is obtainable with the conventional optics.

**Schlieren diffraction interferometer, beam-folding interferometer system and shadowgraph technique**

The philosophy behind the approach of combining different techniques is based on the observation that the inherent shortcomings of individual methods can be compensated in a combination of the techniques\textsuperscript{22–24}. Schlieren diffraction interferometry is simple to realize, but it provides information about transverse phase distributions instead of complete information. Interferometry is highly sensitive, but does not provide accurate information where index gradients are high, which produces path-length variations greater that the wavelength of light used. The high-index gradients could be analysed using the shadowgraph technique. A typical shadowgraph image of the same glass plate obtained in the direct beam is shown in Figure 8. In this technique, information about index gradients becomes available in the form of a shadow pattern of the object that can be obtained on the observation plane OP. To obtain accurate information of the test object, all these techniques could be combined in a single set-up using the multiplexing property of the HOEs. The experimental arrangement for the formation of HOE enabling combined operation of these techniques is schematically shown in Figure 9. Here a collimated reference beam \( R \) is combined with two converging beams propagating along different directions in two holographic exposures on the same recording plate \( H \). After chemical processing, the plate \( H \) is illuminated with reference beam \( R \), which reconstructs two converging beams in the

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**Figure 3.** Typical photograph of infinite-mode, holographic optics-based schlieren diffraction interferometer where a single diffraction fringe covers the field of view.

**Figure 4.** Typical experimental results of schlieren diffraction interferometer on an optical glass plate (\( a \)) with holographic optics and (\( b \)) with conventional optics.

**Figure 5.** Schematic experimental set-up for realization of holographic optics-based beam-folding interferometer.
respective directions. The test object is interposed in the beam $R$ and a schlieren element, i.e. knife-edge or folding mirror positioned at the reconstructed focused spots generate there respective test results while the directly transmitting collimated beam provides the shadowgraphic results.

**Discussion and conclusion**

Realization and operation of holographic optics-based schlieren diffraction interferometer has been described. It has been shown that holographic optics provides the same information that is obtainable with conventional optics and additionally, could combine more than one technique such as schlieren diffraction interferometer, beam-folding interferometer, shadowgraph technique, point-diffraction interferometer, etc. in a single system in order to obtain simultaneously the phase or density gradient distribution in a test field. The proper combination of these techniques can provide an unprecedented wealth of information about phase distribution as each technique has its own characteristic strengths and weaknesses. By applying multiple techniques, the inherent shortcomings of individual methods can be overcome and the risk of overlooking or misinterpreting certain features about the test field is reduced. The proposed schemes could be realized with a single HOE in conjunction with a collimated beam and the focal plane modifying element to carry out test studies on phase objects in real-time. The optical arrangement in the proposed system involves a simple alignment procedure and conventional holographic recording material is used in the formation of HOE. The described method is suitable for industrial applications because a single collimated beam perpendicular to the holographic plate serves as reference as well as test beam, making the system relatively insensitive to external vibrations. Also alignment of the proposed scheme is easy compared to other HOE-based interferometers due to the fact that, by observing and properly adjusting the back-reflected light from the HOE, its position could easily be aligned. This was tested by removing the HOE from the set-up and again aligning it at different locations. It was observed that if the HOE had some misalignment, then the reconstructed focus spot will not be true and it will not be possible to obtain true information about the test object for precise quantitative measurements, but phase visualization could still be performed. The error in retrieved information depends on the amount of misalignment in the repositioning of the HOE. Typical test results of schlieren diffraction interferometer and beam-folding interferometer for a misalignment of 25 $\mu$m are pre-
Figure 10. Typical experimental results with HOE misaligned by 25 μm on the same optical glass plate (a) with schlieren diffraction interferometer and (b) with beam-folding interferometer.

sent in Figure 10a and b respectively. These results show that high contrast fringes are still present, but an error equivalent to misalignment in repositioning of the HOE is introduced. It is known that recording the HOE on silver-halide plates introduces noise in the reconstructed wavefront. This noise could be reduced using dichromatic gelatin (DCG) as the recording material, as the grain size in this case is negligible and DCG gives better efficiency compared to silver-halide recording materials, thereby enhancing the cost-effective hologram-copying methods.


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