Simple lecture demonstration for the nonlinear behavior of absorption in a scattering medium

R. A. Reed
Rt. 2 Box 7, Hillsboro, Tennessee 37342

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Radiative transfer in absorbing and scattering media is often covered in engineering heat transfer courses and it is essential material for advanced students in astronomy and remote sensing. The effects of scattering and, in particular, the nonlinear dependence of reflectivity upon the single scattering albedo, usually comes as a surprise to students. Yet another surprise is just how low the reflectivity can be even for fairly high values of the single scattering albedo. An excellent discussion of these effects, together with a number of delightful and practical applications to the natural world, is given in an article in this Journal by Bohren.[1] The lecture demonstration here uses mixtures of white and black powders to show both the nonlinear dependence of the reflectivity upon the single scattering albedo and the low values of reflectivity corresponding to single scattering albedos in the range near 0.95. An essential part of the demonstration is a deliberately misleading "setup" which adds to the overall interest. (After all, what is so tremendously exciting about mixing black and white together to get gray?)

The relation between the reflectivity R and the single scattering albedo w is first considered for the "setup"—a two-dimensional checkerboard of perfectly black ($w = 0$) and white ($w = 1$) squares. One either moves sufficiently far away from the checkerboard so that the individual squares cannot be resolved or else rapidly rotates the board so as to produce a blurred image. The net reflectivity R is readily understood to be equal to the fraction of the board occupied by white squares, i.e., 50%. The situation for a powder, of course, is quite different. The two-flux approximation, sometimes called the Schuster–Schwarzschild equation,[2] is the simplest solution of the radiative transfer equation that exhibits the correct physical trends. This solution, given as Eq. (28) in Bohren's article, can be readily grasped by advanced undergraduates. The basic point is that, unlike the checkerboard, the reflectivity of an aerosol or powder is a strongly nonlinear function of single scattering albedo for small absorption. In fact, as pointed out in Ref. 1, the derivative of the two-flux reflectivity R with respect to absorption k actually diverges in the limit of small k.

The demonstration itself adds a pinch of dry black pigment powder, such as that used to make children's black tempera paint, to a jar of flour in a proportion of about 1 part to 20. Shake well. The objective is to create a powder with a single scattering albedo in the range of 0.95. The preceding checkerboard "setup" example, where $R = 50\%$ for a half white–half black mixture, leads the uninitiated to expect an almost imperceptible 5% drop in reflectivity. This reasoning fails completely when applied to the demonstration. For my brands of flour (Pillsbury) and pigment (Artista), the reflectivity drops a hefty 25%, from about 85% to 60%. If desired, this can be shown using a photographic gray scale card. The reflectivity of the flour–pigment mixture agrees fortuitously well with the two-flux equation, which predicts a reflectivity of 63% for an isotropically scattering medium with $w = 0.95$. In reality, the scattering from the flour particles is anisotropic and the single scattering albedo of a large carbon particle is 0.5, not 0, but these do not detract from the overall effect. The nonlinearity of the reflectivity can now be demonstrated by adding more pigment (e.g., "How much does the reflectivity drop if the amount of pigment is doubled?" or "How much pigment must be added to halve the reflectivity?") At this point, having aroused curiosity, the physical basis for the nonlinearity, the derivation of the two-flux equations, and a number of practical observations are appropriate. The physical basis for the nonlinearity, as explained in Ref. 1, is the enhancement of absorption by multiple scattering. Presentation of a plot of the photon path length distribution or of the mean number of scatterings versus single scattering albedo, such as Figs. 17.1 and 17.3 of Van de Hulst[3] might be especially helpful at this point. The "setup" example can now be unmasked as an unrealistic two-dimensional model because the black and the white remain unmixed. Practical illustrations of multiple scattering physics abound, including laundry ("Why does a ring-around-the-collar show up so much more readily on a white shirt than on a sporty-looking pastel?"); color control in the carpet, yarn, and fabric industries ("Why does Aunt Matilda insist upon getting all her yarn from the same dye lot?"); and typewriter correction fluid. (Believe it or not, pure correction fluid, composed of MgO, is actually too white. It is necessary to add a trace amount of carbon black to match the reflectivity of ordinary typing paper. Assuming pure MgO is 100% reflective, how much carbon is needed to drop the reflectivity to, say, 95%? How accurately must the carbon black be measured if the tolerance on reflectivity is ± 1%?) On a more amusing note, one might also mention the old proposition (Heaven forbid!) of melting the polar ice caps by the addition of a fine layer of soot. The two-flux equation shows that less soot is needed than might be thought at first, but let's not give any ideas to the Army Corps of Engineers.

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[1] Craig F. Bohren, "Multiple scattering of light and some of its observable consequences," Am. J. Phys. **55**, 524–533 (1987).

[2] A. Schuster, Astrophys. J. **21**, 1 (1905), reprinted in *Selected Papers on the Transfer of Radiation*, edited by D. H. Menzel (Dover, New York, 1966).

[3] H. C. van de Hulst, *Multiple Light Scattering: Tables, Formulas, and Applications* (Academic, New York, 1980).