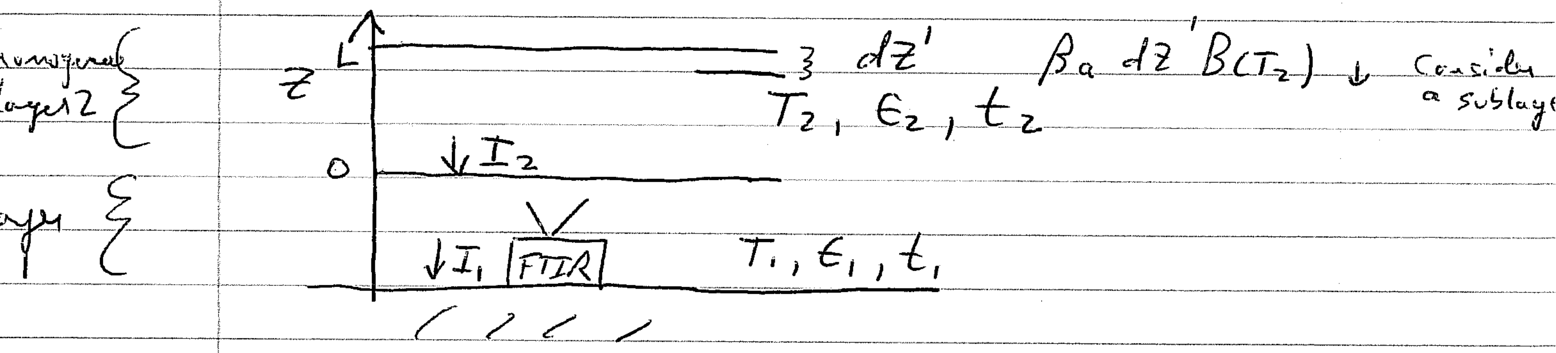
2 layer Idea for FTIR Spectra

(How to understand spectra for an atmosphere with a temperature inversion)



From the Schwarzchild Eq.,

$$t_2 = e^{-\int_0^L \beta_a(z')dz'} = e^{-\beta_{a,2} z'}$$

$$I^{\downarrow}(\nu) = \int_0^{\infty} t(z')\,\beta_a(z')\,B[T(z')]\,dz'$$

For layer 2, parameters are constant, so

$$I_2^{\downarrow} = \beta_a B[T_2] \underbrace{\int_0^L e^{-\beta_a z'}\,dz'}_{-\frac{1}{\beta_a} e^{-\beta_a z'}\Big|_0^L}$$

or

$$I_2^{\downarrow} = B(T_2)\underbrace{\left[1 - e^{-\overbrace{\beta_{a,2}L}^{\tau_2}}\right]}_{\epsilon_2}$$

$$t_2 = e^{-\tau_2} \qquad \epsilon_2 = 1 - t_2$$

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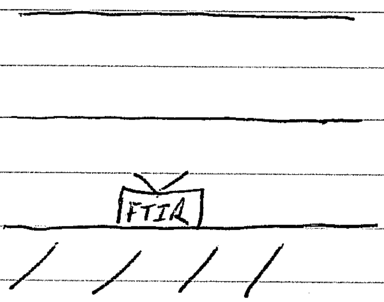
(How to understand spectra for an atmosphere w/ temperature inversion)



Layer 2, T_2, ϵ_2, t_2, $\epsilon_2 = 1 - t_2$

Layer 1, T_1, ϵ_1, t_1, τ_1 $\epsilon_1 = 1 - t_1$ $t_1 = e^{-\tau_1}$

$I = I(\nu)$ = FTIR measurement of ↓ radiance

(1) $$I = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$$

Limits: ① $\epsilon_1 \approx 1$ $t_1 \approx 0$

$I \cong B(T_1)$ regardless of T_2

② $\epsilon_1 \approx \epsilon_2 \approx t_1 \approx t_2$, $T_2 << T_1$

$$I = \epsilon_1 \mathbf{B}(T_1) + \epsilon_2 B(T_2) t_1$$

[Usual temperature profile spectra, $T_2 < T_1$]

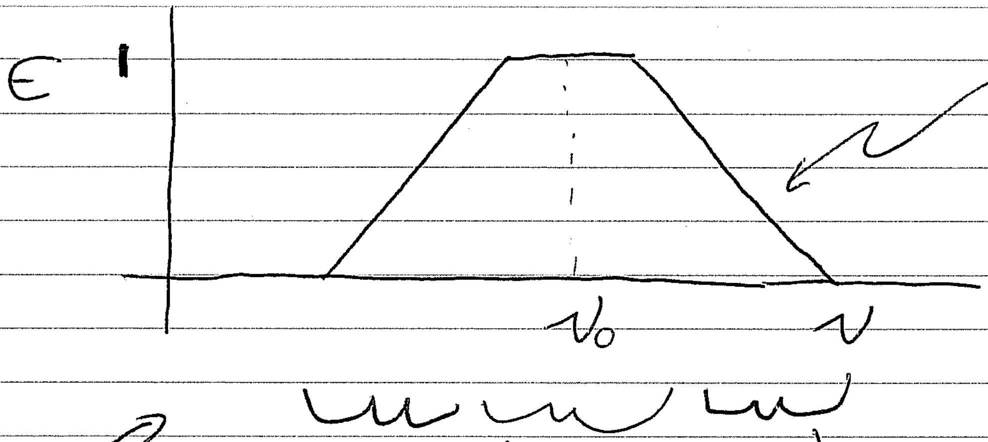
③ $\epsilon_1 \approx \epsilon_2 \approx t_1 \approx t_2$ $T_2 >> T_1$

$$I = \epsilon_1 B(T_1) + \epsilon_2 \mathbf{B}(T_2) t_1$$

[temperature Inversion]

All cases can be explored with a single strong line

Center of absorption line



Wings of absorption line

Wavenumber range where saturated ("See" only layer 1)

Can "see" through layer 1

Generic Shape, Could be region near strong absorption by CO_2 @ ~15 μm.

Does the simple model really work

Exercise: Suppose $T_1 = T_2 = T_0$, then should find

$$I = B(T_0)\,\epsilon_0 \quad , \quad \epsilon_0 = 1 - t_0$$

Show this →

$$t_0 = e^{-\underbrace{(\tau_1+\tau_2)}_{\tau_0}}$$

Proof: $I = \epsilon_1 B(T_1) + \epsilon_2 B(T_2)\, t_1$

$$I = B(T_0)\left[\epsilon_1 + \epsilon_2 t_1\right]$$

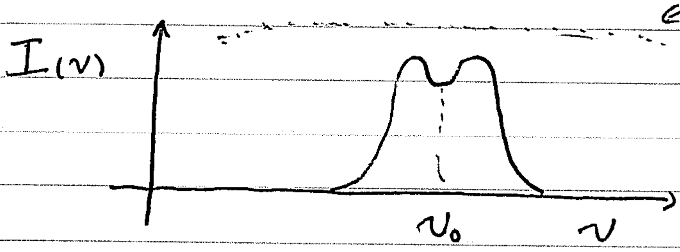
$$= \text{''} \left[1 - t_1 + (1-t_2)t_1\right]$$

$$= \text{''} \left[1 - \cancel{t_1} + \cancel{t_1} - t_1 t_2\right]$$

Since $t_1 t_2 = e^{-\tau_1} e^{-\tau_2} = e^{-(\tau_1+\tau_2)} = e^{-\tau_0}$

$$I = B(T_0)\underbrace{(1 - e^{-\tau_0})}_{\epsilon_0} = B(T_0)\,\epsilon_0$$

Condition for the Schnauzer Profile:



~ Radiance for Blackbody — Sk... va...

For understanding, consider a single strong spectral line at ν_0

ϵ_2, t_2, T_2

$\epsilon_1, t_1, T_1 \downarrow I(\nu) = \epsilon_1 B(T_1) + \epsilon_2 B(T_2) t_1$

$$@\ \nu_0 \quad \epsilon_1 = 1, \quad t_1 = 0 \quad I = B(T_1) = B(T_1, \nu$$

?? Away from ν_0, what are the conditions for $I > B(T_0$

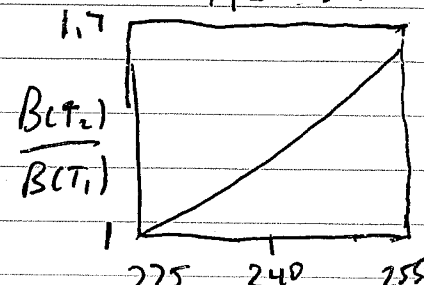
$$\epsilon_1 B(T_1, \nu) + \epsilon_2 B(T_2, \nu) t_1 > B(T_1, \nu_0)$$

Considering $B(T_1, \nu) \simeq B(T_1, \nu_0)$ in local area of ν,

$$\epsilon_2 B(T_2) \cancel{t_1} > B(T_1)(\underbrace{1 - \epsilon_1}_{\cancel{t_1}})$$

or

$$\frac{B(T_2)}{B(T_1)} > \frac{1}{\epsilon_2}$$



$\nu = 667\ cm^{-1}$
$\lambda = 15\ \mu m$

Could solve by using

$$\frac{B(T_2)}{B(T_1)} = \frac{e^{hc\nu/k_B T_1} - 1}{e^{hc\nu/k_B T_2} - 1},$$