Lecture 13

Light scattering and absorption by atmospheric particulates. Part 1: Main concepts: elementary wave, polarization, Stokes matrix, and scattering phase function. Rayleigh scattering.

Objectives:

1. Main concepts: elementary wave, polarization, Stoke matrix, and scattering phase function.
2. Rayleigh scattering.

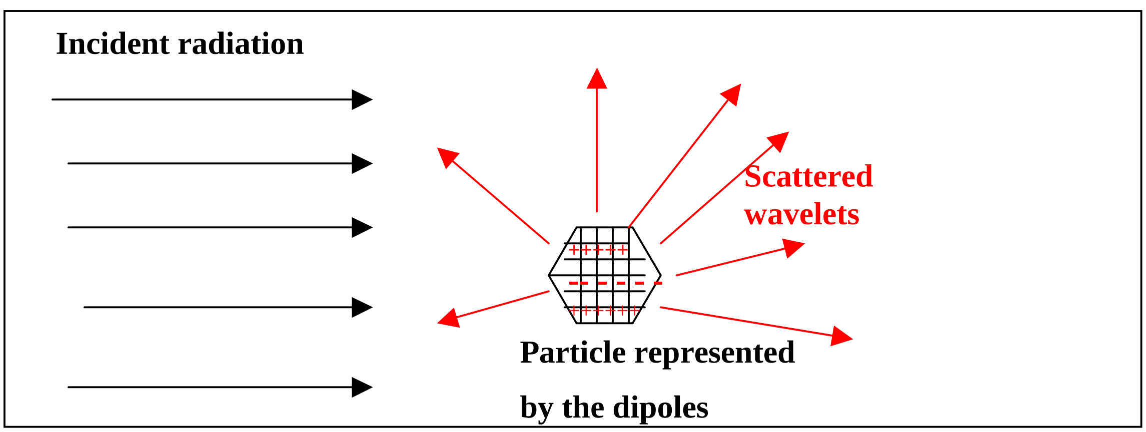
Required Reading:

L02: 3.3.1, 5.3

Additional/advanced Reading:

Bohren, G.F., and D.R. Huffmn, Absorption and scattering of light by small particles. John Wiley&Sons, 1983.

1. Main concepts: elementary wave and light beam, polarization, Stoke matrix, and scattering phase function.



Consider a single arbitrary particle. The incident electromagnetic field induces dipole oscillations. The dipoles oscillate at the frequency of the incident field and therefore scatter radiation in all directions. In a given direction of observation, the total scattered field is a superposition of the scattered wavelets of these dipoles. Scattering can be considered as two step process: (1) excitation and (2) reradiation.

- Scattering of the electromagnetic radiation is described by **the classical electromagnetic theory**, considering the propagation of a light beam as a transverse wave motion (collection of **electromagnetic individual waves)**.

- Electromagnetic field is characterized by the **electric vector** $\vec{E}$ and **magnetic vector** $\vec{H}$, which are orthogonal to each other and to the direction of the propagation. $\vec{E}$ and $\vec{H}$ obey the **Maxwell equations** (see Lecture 14).

Poynting vector gives the flow of radiant energy and the direction of propagation as (in cgs system)

$$\vec{S} = \frac{c}{4\pi} \vec{E} \times \vec{H} \qquad [13.1]$$

$\vec{S}$ is in units of energy per unit time per unit area (i.e. flux);

NOTE: $\vec{E} \times \vec{H}$ means a **vector product** of two vectors.

Thus

$$I = \frac{1}{\Delta\Omega} \frac{c}{4\pi} |E|^2$$

- Since electromagnetic field has wave-like nature, the classical theory of wave motion is used to characterize the propagation of radiation.

Consider a ***plane wave*** propagating in z-direction (i.e., E oscillates in the x-y plane).

The electric vector $\vec{E}$ may be decomposed into the parallel E_l and perpendicular E_r components, so that

$$E_l = a_l \exp(-i\delta_l) \exp(-ikz + i\omega t) \qquad [13.2a]$$

$$E_r = a_r \exp(-i\delta_r) \exp(-ikz + i\omega t) \qquad [13.2b]$$

where $\boldsymbol{a_l}$ and $\boldsymbol{a_r}$ are the **amplitude** of the parallel E_l and perpendicular E_r components, respectively; $\boldsymbol{\delta_l}$ and $\boldsymbol{\delta_r}$ are the **phases** of the parallel E_l and perpendicular E_r components, respectively; $\boldsymbol{k}$ is the propagation (or wave) constant, $\boldsymbol{k = 2\pi/\lambda}$, and $\boldsymbol{\omega}$ is the circular frequency, $\omega = \boldsymbol{kc=2\pi c/\lambda}$

Eq.[13.2] can be written in cosine representation as

$$E_l = a_l \cos(\zeta + \delta_l)$$

$$E_r = a_r \cos(\zeta + \delta_r)$$

where $\zeta = kz - \omega t$ and $\zeta+\delta$ is called **phase**.

Then we have

$$E_l / a_l = \cos(\zeta)\cos(\delta_l) - \sin(\zeta)\sin(\delta_l) \qquad [13.3]$$

$$E_r / a_r = \cos(\zeta)\cos(\delta_r) - \sin(\zeta)\sin(\delta_r)$$

and thus

$$(E_l / a_l)^2 + (E_r / a_r)^2 - 2(E_l / a_l)(E_r / a_r)\cos(\delta) = \sin^2(\delta) \qquad [13.4]$$

where $\delta = \delta_r - \delta_l$ is the **phase difference** (or **phase shift**).

Eq.[13.4] represents an ellipse => **elliptically polarized wave**

If $\delta = m\pi$ (m =0, +/1; +/-2…), then $\sin(\delta)$ =0 and Eq.[13.4] becomes

$$\left(\frac{E_l}{a_l} \pm \frac{E_r}{a_r}\right)^2 = 0 \quad \text{or} \quad \frac{E_l}{a_l} = \pm\frac{E_r}{a_r} \qquad [13.5]$$

Eq.[13.5] represents two perpendicular lines => **linearly polarized wave**

If $\delta = m\pi/2$ (m = +/1; +/-2…) and $a_l = a_r = a$, Eq.[13.4] becomes

$$E_l^2 + E_r^2 = a^2 \qquad [13.6]$$

Eq.[13.6] represents a circle => **circularly polarized wave**

- In general, light is a superposition of many waves of different frequencies, phases, and amplitudes. Polarization is determined by the relative size and correlations between two electrical field components. Radiation may be unpolarized, partially polarized, or completely polarized.
- **Natural light (sunlight) is unpolarized.**
- If there is a definite relation of phases between different scatterers => radiation is called **coherent.** If there is no relations in phase shift => light is called **incoherent**
- **Natural light (sunlight) is incoherent.**

The property of incoherent radiation:

The intensity due to all scattering centers is the sum of individual intensities.

NOTE: In our course, we study the **incoherent scattering of the atmospheric radiation.**

NOTE: The assumption of independent scatterers is violated if the particles are too closely packed (spacing between particles should be several times their diameters to prevent intermolecular forces from causing correlation between scattering centers).

- Eq.[13.4] shows that, in the general case, three independent parameters ***a_l, a_r*** and δ are required to characterize an electromagnetic wave. These parameters are not measured. Therefore, a new set of parameters (which are proportional to intensity) has been proposed by Stoke.

Stokes parameters: so-called intensity I, the degree of polarization Q, the plane of polarization U, and the ellipticity V of the electromagnetic wave

$$I = E_l E_l^* + E_r E_r^*$$

$$Q = E_l E_l^* - E_r E_r^* \qquad [13.7]$$

$$U = E_l E_r^* + E_r E_l^*$$

$$V = -i(E_l E_r^* - E_r E_l^*)$$

They are related as

$$I^2 = Q^2 + U^2 + V^2 \qquad [13.8]$$

Stokes parameter can be also expressed as

$$I = a_l^2 + a_r^2$$

$$Q = a_l^2 - a_r^2 \qquad [13.9]$$

$$U = 2 a_l a_r \cos(\delta)$$

$$V = 2 a_l a_r \sin(\delta)$$

- Actual light consists of **many individual waves** each having its own amplitude and phase.

NOTE: During a second, a detector collects about millions of individual waves. Measurable intensities are associated with the superposition of many millions of simple waves with independent phases. Therefore, for a light beam the Stokes parameters are averaged over a time period and may be represented as

$$\begin{aligned} I &= \langle a_l^2 \rangle + \langle a_r^2 \rangle = I_l + I_r \\ Q &= \langle a_l^2 \rangle - \langle a_r^2 \rangle = I_l - I_r \\ U &= \langle 2 a_l a_r \cos(\delta) \rangle \\ V &= \langle 2 a_l a_r \sin(\delta) \rangle \end{aligned} \qquad [13.10]$$

where $\langle .... \rangle$ denote the time averaging.

For a light beam, we have

$$I^2 \geq Q^2 + U^2 + V^2 \qquad [13.11]$$

The **degree of polarization** ***DP*** of a light beam is defined as

$$DP = (Q^2 + U^2 + V^2)^{1/2} / I \qquad [13.12]$$

The **degree of linear polarization** ***LP*** of a light beam is defined by neglecting U and V as

$$LP = -\frac{Q}{I} = -\frac{I_l - I_r}{I_l + I_r} \qquad [13.13]$$

Unpolarized light: $Q = U = V = 0$

Fully polarized light: $I^2 = Q^2 + U^2 + V^2$

Linear polarized light: $V = 0$

Circular polarized light: $|V| = I$

➢ The **scattering phase function P(cosΘ)** is defined as a non-dimensional parameter to describe the angular distribution of the scattered radiation as

$$\boxed{\frac{1}{4\pi}\int_{\Omega} P(\cos\Theta)\, d\Omega = 1} \qquad [13.14]$$

where Θ is called the **scattering angle** between the direction of incidence and observation.

NOTE: The phase function is expressed as

$$P(\cos\Theta) = P(\theta', \varphi', \theta, \varphi),$$

where **(θ', φ')** and **(θ, φ)** are the spherical coordinates of incident beam and direction of observation, and (see L02: Appendix C):

$$\cos(\Theta) = \cos(\theta')\cos(\theta) + \sin(\theta')\sin(\theta)\cos(\varphi'-\varphi) \qquad [13.15]$$

Forward scattering refers to the observations directions for which $\Theta < \pi/2$

Backward scattering refers to the observations directions for which $\Theta > \pi/2$

2. Rayleigh scattering

Consider a small homogeneous spherical particle (e.g., molecule) with size smaller than the wavelength of incident radiation $\vec{E}_0$. Then the induced dipole moment $\vec{p}_0$ is

$$\vec{p}_0 = \alpha \vec{E}_0 \qquad [13.16]$$

where **α** is the **polarizability** of the particle.

NOTE: Do not confuse the polarization of the medium with polarization associated with the EM wave.

According to the classical electromagnetic theory, the scattered electric field at the large distance ***r*** (called far field scattering) from the dipole is given (in cgs units) by

$$\vec{E} = \frac{1}{c^2}\frac{1}{r}\frac{\partial \vec{p}}{\partial t}\sin(\gamma) \qquad [13.17]$$

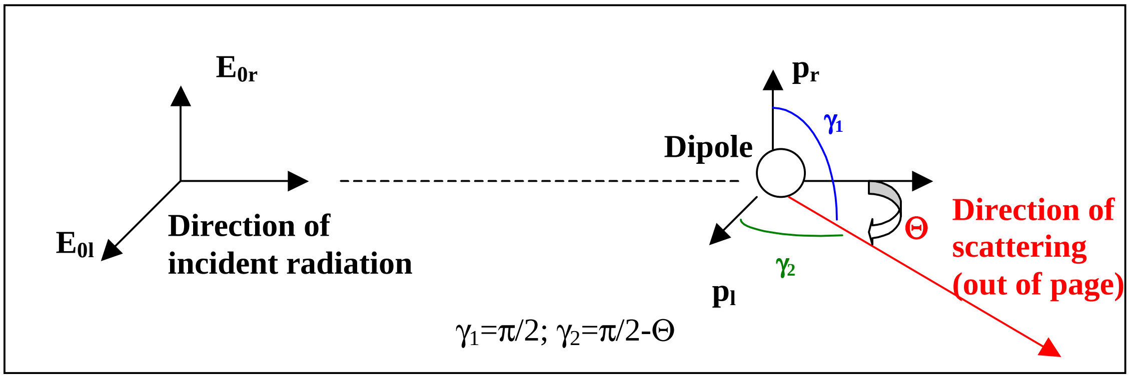
where **γ** is the angle between the scattered dipole moment $\vec{p}$ and the direction of observation.

In an oscillating periodic field, the dipole moment is given in terms of induced dipole moment by

$$\vec{p} = \vec{p}_0 \exp(-ik(r - ct)) \qquad [13.18]$$

and thus the electrical field is

$$\vec{E} = -\vec{E}_0 \frac{\exp(-ik(r - ct))}{r} k^2 \alpha \sin(\gamma) \qquad [13.19]$$



NOTE: **Plane of scattering** (or **scattering plane**) is defined as a plane containing the incident beam and scattered beam in the direction of observation.

Decomposing the electrical vector on two orthogonal components perpendicular and parallel to the plane of scattering (a plane containing the incident and scattering beams), We have

$$E_r = -E_{0r} \frac{\exp(-ik(r - ct))}{r} k^2 \alpha \sin(\gamma_1) \qquad [13.20]$$

$$E_l = -E_{0l} \frac{\exp(-ik(r - ct))}{r} k^2 \alpha \sin(\gamma_2)$$

Using that

$$I = \frac{1}{\Delta\Omega} \frac{c}{4\pi} |E|^2, \qquad [13.21]$$

perpendicular and parallel intensities (or linear polarized intensities) are

$$I_r = I_{0r} k^4 \alpha^2 / r^2 \qquad [13.22]$$

$$I_l = I_{0l} k^4 \alpha^2 \cos^2(\Theta) / r^2$$

Using that the natural light (incident beam) in not polarized (***I*₀ᵣ**) and that

k=2π/λ, we have

$$\boxed{I = I_r + I_l = \frac{I_0}{r^2}\alpha^2\left(\frac{2\pi}{\lambda}\right)^4 \frac{1+\cos^2(\Theta)}{2}} \qquad [13.23]$$

Eq.[13.23] gives the intensity scattered by molecules for unpolarized incident light, Rayleigh scattering.

Rayleigh scattering phase function for incident unpolarized radiation (follows from Eq.[13.23]) is

$$\boxed{P(\cos(\Theta)) = \frac{3}{4}(1+\cos^2(\Theta))} \qquad [13.24]$$

Eq.[13.23] may be rewritten in the form

$$\boxed{I(\cos(\Theta)) = \frac{I_0}{r^2}\alpha^2 \frac{128\pi^5}{3\lambda^4}\frac{P(\Theta)}{4\pi}} \qquad [13.25]$$

Eq.[13.23] may be rewritten in the terms of the scattering cross section

$$\boxed{I(\cos(\Theta)) = \frac{I_0}{r^2}\sigma_s \frac{P(\Theta)}{4\pi}} \qquad [13.26]$$

Here the scattering cross section (in units or area) by a single molecule is

$$\boxed{\sigma_s = \alpha^2 \frac{128\pi^5}{3\lambda^4}} \qquad [13.27]$$

The polarizability $\boldsymbol{\alpha}$ is given by the Lorentz-Lorenz formula (see L02: Appendix D):

$$\alpha = \frac{3}{4\pi N_s}\left(\frac{m^2-1}{m^2+2}\right) \qquad [13.28]$$

where $\boldsymbol{N}$ in the number of molecules per unit volume and $\boldsymbol{m=m_r+im_i}$ in the refractive index.

NOTE: For air molecules in solar spectrum $\boldsymbol{m_r}$ is about 1 but depends on λ, and $\boldsymbol{m_i}$ =0 (see Lecture 14).

Thus the polarizability can be approximated as

$$\alpha \approx \frac{1}{4\pi N_s}(m_r^2 - 1) \qquad [13.29]$$

Therefore the scattering cross section of air molecules (Eq.[13.27]) becomes

$$\boxed{\sigma_s = \frac{8\pi^3 (m_r^2 - 1)^2}{3\lambda^4 N_s^2} f(\delta)} \qquad [13.30]$$

where ***f*(δ)** is the correction factor for the anisotropic properties of air molecules, defined as ***f*(δ) =(6+3δ)/(6-7δ) and δ=0.035**

Using this scattering cross section, one can calculate *the optical depth of the entire atmosphere due to molecular scattering* as

$$\boxed{\tau(\lambda) = \sigma_s(\lambda)\int_0^{top} N(z)dz} \qquad [13.31]$$

Approximation of molecular Rayleigh optical depth (i.e., optical depth due to molecular scattering) down to pressure level p in the Earth's atmosphere:

$$\tau(\lambda) \approx 0.0088\left(\frac{p}{1013\ mb}\right)\lambda^{-4.15+0.2\lambda} \qquad [13.32]$$

Rayleigh scattering results in the sky polarization. The degree of linear polarization is

$$LP(\Theta) = -\frac{Q}{I} = -\frac{I_l - I_r}{I_l + I_r} = \frac{\cos^2\Theta - 1}{\cos^2\Theta + 1} = \frac{\sin^2\Theta}{\cos^2\Theta + 1} \qquad [13.33]$$

Forward and backward scattering direction: unpolarized light

90^o scattering angle: completely polarized