Optical properties of terrestrial clouds

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Abstract

The aim of this review is to consider optical characteristics of terrestrial clouds. Both single and multiple light scattering properties of water clouds are studied. The numerous results discussed can be used for solutions both inverse and direct problems of the cloud optics.

CONTENTS

1. Introduction

Water, ice, and mixed clouds are major regulators of solar fluxes in the Earth atmosphere(Kondratyev and Binenko, 1984; Liou, 1992). They reflect a great portion of incoming visible radiation back to outer space. The light energy absorbed by water droplets and ice crystals leads to the heating of atmospheric layers. Another interesting role of the cloudy media is to serve as a blanket to protect the Earth surface against cooling at night. This is due to the fact that the maximum of the terrestrial emission is in the far infrared, where water droplets are highly absorbing.

Clouds, mists, and fogs are very common. They reduce the visibility of objects in the atmosphere and limit capabilities of atmospheric vision, remote sensing and detection systems(Zege et al., 1991). Thus, it is of a great importance to understand the peculiarities of light interaction with cloudy media, which can be considered as a huge ’’colloid ’’ , composed of liquid and solid water particles, dispersed in the air. It should be pointed out that water droplets and air around them can be contaminated by various fine particles(e.g., soot and dust particles). These aerosol particles influence both light scattering and absorption properties of cloudy media, making the studies of problem of light propagation in cloudy media even more difficult(Menon et al., 2001). Note, that the very existence of clouds is due to fine aerosol particles, so called cloud nuclei(Twomey, 1977). The existence of clouds would be not possible in the environment, which does not have these fine aerosol nuclei, with typical radii between 5 and 200 nm (Mason, 1975).

Studies of optical and microphysical properties of cloudy media have a long and fruitful history. The results of these investigations have been summarized in numerous books and papers(Kondratyev and Binenko, 1984; Zege et al., 1991; Liou, 1992; Kokhanovsky, 2001a). We can state that the main properties of cloudy media are well understood now. However, this is not the end of the story. There are a lot unsolved problems. The most important are the account for the three dimensional shape of clouds(Macke et al., 1999, Scheirer and Macke, 2000; Scheirer, 2001), their inhomogeneity in horizontal and vertical directions(Cahalan et al., 1994, 2001; Platnick, 2000, 2001). The influence of aerosols presented inside cloudy media, which can be one of explanations of the anomalous absorption paradox (Danielson et al., 1968; Rozenberg et al., , 1978), should be also clarified.

Another hot issue is the characterization of optical properties of ice clouds, which have extremely complex microstructure(Liou, 1992; Macke et al., 1996; Yang and Liou, 1998; Yang et al., 2000,2001) and appear almost with the same frequency as water clouds. One can hardly find two identical crystals in ice clouds, which often present in the terrestrial atmosphere. Preferential shapes vary with temperature and pressure(Mason, 1975). Thus, they are different in different regions of cloudy media(Yang et al., 2001). It should be stressed that optical properties of a single particle are extremely influenced by its shape (Mishchenko et al., 1995, 1996, 1999, 2000). And so do the optical properties of a cloud as a whole.

It is virtually impossible to make a complete review across all fields of cloud optics, which is in a condition of a continious explosion , even in a book, not mention an article. Thus, I will concentrate mostly on analytical results, which can be used for cloud optical properties studies. The formulae presented can be applied for rapid estimations of light fluxes in cloudy atmosphere. They are also of help in understanding the information content of transmitted and reflection functions of cloudy media in respect to the microstructure parameters of clouds .

2. Microphysics of clouds

2.1 Water clouds

2.1.1 Particle size distributions

Water clouds consist of small liquid droplets, which generally have the spherical shape. Particles of other shapes can appear due to different external influences. For instance, the deformation of large particles due to the gravitation force is of importance for raining clouds with particles having radii 1 *mm* and larger(Macke and Grossklaus, 1998). The average radius of droplets in non-raining water clouds is usually around 10 micrometers and the approximation of spherical particles works quite well. Natural clouds with droplets of the same fixed size throughout its volume never occur due to the variability of physical properties of the atmospheric air both in space and time domains(Twomey, 1977). Thus, one can consider a radius of a droplet a as a random value, which is characterized by the distribution function $f(a)$. This function is normalized by the following condition:

$$\int_0^\infty f(a)da = 1. \tag{2.1}$$

It should be pointed out that the integral

$$F(a) = \int_{a_1}^{a_2} f(a)da \tag{2.2}$$

gives the fraction of particles with radii between a_1 and a_2 in a unit volume of a cloud. The distribution function $f(a)$ can be represented as a histogram, graphically or in a tabular form (Ayvazian, 1988). However, it is the most common to use an analytical form of this function. Analytical functions involving only two parameters are generally used (Deirmendjian, 1969). This is, of course, a great simplification of real situations, which occur in natural clouds. However, it was found that most of optical characteristics of a cloud as a whole practically do not depend on the fine structures of particle size distributions (PSD) $f(a)$. Even specific types of analytical functions used could be of a minor importance (Hansen and Travis, 1974) in most of cases.

It was found (McGraw et al., 1998) that the local optical properties of polydispersions can be modeled with high accuracy by just first six moments of the particle size distribution. The use of certain combination of moments can reduce the number of parameters involved even further.

In most of cases the function $f(a)$ can be represented by the gamma distribution:

$$f(a) = Aa^{\mu} e^{-\mu \frac{a}{a_0}}, \tag{2.3}$$

where

$$A = \frac{\mu^{\mu+1}}{\Gamma(\mu+1) a_0^{\mu+1}} \tag{2.4}$$

is the normalization constant and $\Gamma(\mu+1)$ is the Gamma function. It follows that $f'(a_0) = 0$ and $f''(a_0) < 0$. Thus, the function (3) has the maximum at $a = a_0$. Note, that Eq. (2.4) follows from Eq. (2.1) and the definition of the Gamma function:

$$\Gamma(\mu) = \int_0^{\infty} x^{1-\mu} e^{-x} dx\,. \tag{2.5}$$

One can see that the parameter μ characterizes the width of the particle size distribution $f(a)$. It is smaller for wider distributions. Moments

$$\langle a^n \rangle = \int_0^{\infty} a^n f(a) da \tag{2.6}$$

of the distribution (2.3) are calculated from the following simple equation

$$\langle a^n \rangle = \left(\frac{a_0}{\mu} \right)^n \frac{\Gamma(\mu+n+1)}{\Gamma(\mu+1)}\,. \tag{2.7}$$

Eq. (2.7) can be used to find the average volume of particles

$$\langle V \rangle = \frac{4\pi}{3} \int_0^{\infty} a^3 f(a) da\,, \tag{2.8}$$

the average surface area

$$\langle \Sigma \rangle = 4\pi \int_0^{\infty} a^2 f(a) da\,, \tag{2.9}$$

the average mass of droplets

$$\langle W \rangle = \rho \langle V \rangle\,, \tag{2.10}$$

where $\rho = 10^6\, g/m^3$ is the density of water, and other important physical characteristics. Namely , it follows:

$$\langle V \rangle = \frac{\Gamma(\mu+4)}{\mu^3 \Gamma(\mu+1)} v_0 , \tag{2.11}$$

$$\langle \Sigma \rangle = \frac{\Gamma(\mu+3)}{\mu^2 \Gamma(\mu+1)} s_0 , \tag{2.12}$$

$$\langle W \rangle = \frac{\Gamma(\mu+4)}{\mu^3 \Gamma(\mu+1)} w_0 , \tag{2.13}$$

where

$$v_0 = \frac{4\pi a_0^3}{3},\ \mathrm{s}_0 = 4\pi a_0^2,\ w_0 = \rho v_0 \tag{2.14}$$

are correspondent parameters for monodispersed particles with radii a_0. One can obtain in the case of the most often employed cloud model with $a_0 = 4\mu m$, $\mu = 6$ (Cloud C.1 model of Deirmendjian, 1969):

$$< V >= \frac{7}{3} v_0,\ <\Sigma>= \frac{14}{9} s_0,\ < W >= \frac{7}{3} w_0 \tag{2.15}$$

where $v_0 \approx 2.7\times10^{-16}\,\mathrm{m}^3, s_0 \approx 2\times10^{-12}\,\mathrm{m}^2, w_0 \approx 2.7\times10^{-10}\, g$. One can see how small these numbers are. What makes cloud droplets so important in atmospheric studies are their numbers(see next Section).

Eq. (2.3) allows to characterize the cloud microstructure only by two parameters: a_0 and μ. However, it should be remembered that both of them are not constant and can vary inside the body of a cloud. Thus, their values depend on the averaging scale. Naturally, large averaging scales produce more broad particle size distributions with smaller values of μ. The value of μ = 2 was found to be rather representative in this case (Khrgian and Mazin, 1952). Thus, this value is advised to be used in low resolution cloud satellite retrieval algorithms. The parameter μ = 6 (Deirmendjian, 1969) , used in the derivation of Eq. (2.15) , is typical only for small averaging scales (Fomin and Mazin, 1998). The general features of the droplet spectrum in water clouds were studied in detail by Warner(1973).

The parameters a_0 and μ are defined in terms of the specific uni-modal cloud droplet distribution (2.3). However, there is a necessity for characterizing cloud particle size distributions by their moments, which can be retrieved from optical measurements, not referring to specific distributions (McGraw et al., 1998). It was found (Hansen and Travis, 1974) that the effective radius

$$a_{ef} = \frac{\langle a^3 \rangle}{\langle a^2 \rangle} \tag{2.16}$$

is one of the most important parameters of any particle size distribution. It is proportional to the average volume/surface ratio of droplets. The parameter (2.16) can be also defined for nonspherical particles as we will see later. The coefficient of variance (CV) of the particle size distribution

$$\Delta = \frac{s}{\langle a \rangle} \tag{2.17}$$

where

$$s = \sqrt{\int_0^\infty \left(a - \langle a \rangle\right)^2 f(a)da}\ , \tag{2.18}$$

is also of importance (especially for narrow droplet distributions). The value of s is called the standard deviation. Thus, the CV is equal to the ratio of the standard deviation to the mean radius $\langle a \rangle$. It is often expressed in percent. It follows for the PSD (2.3):

$$a_{ef} = a_0\left(1 + \frac{3}{\mu}\right),\ \Delta = \frac{1}{\sqrt{1+\mu}} \tag{2.19}$$

or

$$\mu = \frac{1}{\Delta^2} - 1,\ a_0 = \frac{1-\Delta^2}{1+2\Delta^2} a_{ef}\ . \tag{2.20}$$

For instance, it follows at μ = 3: $a_{ef} = 2a_0$, $\Delta = 0.5$, $s = \langle a \rangle / 2$. One can see that the effective radius is always larger than the mode radius a_0.

Eq. (2.20) gives the meaning of the parameter μ in the PSD (2.3). In situ measurements show that the value of a_0 often varies from 4 to 20 micrometers (Mason, 1975) and $\mu \in [2,8]$ in most of cases. It should be pointed out that clouds with smaller droplets are not stable due to the coagulation and condensation processes (Twomey, 1977). Particles with a large radius can not reside in atmosphere for a long time due to the gravitation force. Thus, several physical processes lead to the existence of the most frequent mode radius range. One can obtain from Eq. (2.19) and inequality $2 \le \mu \le 8$ that the value of $\Delta \in [0.3, 0.6]$. Thus, it follows that the standard deviation of the radius of particles in water droplets is from 30 to 60 percent of an average radius in most of cases. Smaller and larger values of Δ do occur sometimes. However, values of Δ smaller than 0.1 were never observed (Twomey, 1977). Larger values of Δ may indicate the presence of the second mode in the range of large particles(Ayvazian, 1988).

Eq. (2.19) and results for a_0 and μ just reported lead to the effective radius a_{ef} of water droplets in the range from 5 to 50 micrometers, depending on the cloud type. Near-global survey of the value of a_{ef}, using satellite data, shows that typically $5\mu m \le a_{ef} \le 15\mu m$ (Han et al. 1994). We see that water clouds with $a_{ef} > 15\mu m$ is of a rare occurrence. This is can be used to discriminate pixels with ice crystals even at wavelengths, where ice and water absorption coefficients are of about the same size. Such a possibility of discrimination is due to much larger (e.g., in 5-10 times) effective sizes of ice crystals as compared to droplets. The large size of ice crystals will reduce the reflection function in near infrared considerably as compared to droplets. This reduction can be easily detected.

In conclusion, it should be pointed out that some authors prefer to use the representation of the particle size distribution by the following analytical form(Ayvazian, 1988):

$$f(a) = \frac{1}{\sqrt{2\pi}\sigma a} \exp\left(-\frac{\ln^2 \frac{a}{a_m}}{2\sigma^2} \right) \qquad (2.21)$$

which is called the log-normal distribution. The relations between values of a_{ef}, $\langle a \rangle$, Δ and parameters of the gamma and lognormal particle size distributions are presented in Table 1. The last column of this Table represents the effective variance, defined as

$$\Delta_{ef} = \frac{\int_0^\infty \left(a - a_{ef}\right)^2 a^2 f(a) da}{a_{ef}^2 \int_0^\infty a^2 f(a) da}.$$

This value is often used instead of the coefficient of variance Δ (Hansen and Travis, 1974). This is mostly due to a special importance, attached to the value of the effective radius of droplets a_{ef} in a cloudy medium as compared to the average radius $<a>$ of droplets. For instance, light extinction in clouds is governed mostly by values of a_{ef} and liquid water content independently on the particle size distribution.

2.1.2. The concentration of droplets

Clearly, not only the size of particles but also the number concentration of droplets N is of importance for the optical waves propagation, scattering, and extinction in cloudy media. The concentration of droplets depends on the concentration C_N of atmospheric condensation nuclei in the atmospheric air. The value of C_N is smaller over oceans than over continents. Thus, the concentration of droplets in marine clouds are in average larger then in the case of water clouds over continents. Generally, the smaller concentration of droplets N means that they can grow larger, producing clouds with larger droplets over oceans(Han et al., 1994). Svensmark and Friis – Christiansen (1997) and Marsh and Svensmark(2000) have speculated that cosmic ray ionization could influence the production of condensation nucleii and, therefore, cloud properties(Swensmark, 1998). The dimensionless volumetric concentration of droplets $C_v = N\langle V\rangle$ and liquid water content (LWC) $C_w = \rho C_v$ or (see Eq. (2.10)) $C_w = N\langle W\rangle$ are often used in cloud studies as well.

The typical variability of N, C_v, and C_w in water clouds is presented in Table 2 (Mason, 1975; Fomin and Mazin, 1998). It should be remembered that these values can change in broader range in real situations. Thus, numbers in Table 2 are given only for a general orientation. The liquid water content is not constant through out the body of a cloud. It has larger values near the tops of clouds(Feigelson, 1981; Paul, 2000; Yum and Hudson, 2001).

The liquid water path (LWP) w is defined as

$$w = \int_{z_1}^{z_2} C_w(z)dz\,, \qquad (2.22)$$

where $l = z_2 - z_1$ is the geometrical thickness of a cloud. It follows at $C_w = const$:

$$w = C_w l \qquad (2.23)$$

The geometrical thickness of clouds varies, depending on the cloud type (Landolt-Bornstein, 1986). Usually it is in the range 500 - 1000*m* for stratocumulus clouds. Near global data, obtained by Han et al. (1994) from satellite measurements show that the liquid water path is typically in the range 50-150 g/m^2 . The annual mean is equal to 86 g/m^2 (Han et al., 1994).

Cloud systems can easily cover area $s \approx 10^3\,km^2$ (Kondratyev and Binenko, 1984) The total amount of water $w_t = ws$ stored in such a water cloud system is equal to $10^8\,kg$ at $w = 100\,g/m^2$. This underlines the importance of clouds both for climate problems and human activity(e.g., crops production, etc.).

2.1.3. Refractive index of liquid and frozen droplets.

The complex refractive index of particles suspended in the atmosphere is another important parameter in atmospheric optics studies(Liou, 1992). This is due to the fact that not only the size of particles, their shape and concentration influence the light propagation in atmosphere. The chemical composition and thermodynamic phase of particles is of importance as well. The refractive index of water droplets and ice crystals varies with the temperature. It is tabulated by many authors (see, e.g., Halle and Querry (1973) for liquid water and Warren(1984) for ice).

Spectral dependencies of real and imaginary parts of the complex refractive index of water and ice are presented in Figs. 1, 2. The differences in light absorption by liquid and solid water are considerable. This fact can be used for the retrieval of the cloud thermodynamic phase(liquid, ice or mixed phase clouds) from satellite measurements. In particular, we see that the absorption band around the wavelength $\lambda = 1.55\,\mu m$ for ice is shifted to larger wavelengths as compared to liquid water. This shift in the absorption band position can be easily detected with the use of modern spectrometers(Bovensmann et al., 1999).

It follows that the real part of the refractive indices of water n_w and ice n_i do not vary considerably in the visible and near-infrared regions of electromagnetic spectra. One can see that generally $n_i(\lambda) < n_w(\lambda)$. The value n_w is in the range 1.33 - 1.34 for $\lambda = 0.4 - 1\mu m$ and it is in the range 1.30 - 1.32 for ice within the same spectral band. Larger values of refractive indices occur at shorter wavelengths.

The spectral variability of the imaginary part of the refractive index of water, which is responsible for the level of absorption of solar radiation by clouds , is much higher(see Fig.2). It changes six orders of magnitude in the spectral range $0.4 - 2\mu m$ both for liquid water and ice.

It should be pointed out that different impurities in water droplets (mainly soot (Markel, 2002) and various aerosol particles (Twomey, 1977)) can change the imaginary part of the refractive index of droplets (especially in visible, where water is almost transparent). This can influence the accuracy of modern cloud remote sensing techniques(Nakajima et al., 1991; Schuller et al., 2000).

2.2. Ice clouds

Microphysical properties of crystal clouds can not be characterized by a single particle size distribution curve as it is in the case of liquid clouds even if one considers relatively small volumes of a cloudy medium. This is due to extremely complex shapes of ice particles in crystals clouds. Major shapes of ice crystals are plates, columns, needles, sheaths, dendrites, stellars and bullets(Mason, 1975; Liou, 1992; Yang et al., 2001). Combinations of bullets and needles are also common. The Magano-Lee classification of natural crystals includes 80 shapes(Magano and Lee, 1966), ranging from the elementary needle (the classification index N1a) to the irregular germ (the classification index G6).

The concentration of crystals *N* varies with height. It is often in the range 50-50000 crystals per cubic meter . The ice water content

$$C_s = N\langle W \rangle, \tag{2.24}$$

where $\langle W \rangle$ is average mass of crystals, is usually in the range $10^{-4} - 10^{-1}\, g/m^3$. Thus, the average mass of a crystal is in the range $2 \cdot 10^{-3} - 2 \cdot 10^{-9}$ gram. Note that most of crystals have a bulk density ρ, which is less than that of the bulk ice ($\rho = 0.3 - 0.9\, g/cm^3$). This is due the presence of impurities and bubbles inside ice particles(Landolt-Bernstein, 1988). The size of crystals is usually characterized by their maximal dimension *D*. It is usually in the range 0.1 - 6*mm* for single crystals and 1 - 15*mm* for snow crystal aggregates. The smaller crystals(e. g., with maximal sizes around 20 micrometers), however, also present in ice clouds (Yang et al., 2001). The mode D_0 of distribution curves $f(D)$ depends on the shape of crystals. Characteristic values of D_0 are 0.5*mm* for plates and columns, 1*mm* for needles, sheaths and stellars, and 2*mm* for dendrites. Distribution curves can be modeled by gamma distributions with different half-widths. Half-widths of distributions $\Delta_{1/2}$ are larger for larger values of D and often it holds: $\Delta_{1/2} \approx \bar{D}$, where $\bar{D}$ is the average size of crystals.

Simple shapes of ice crystals (e.g., hexagonal prisms) can be characterized by two dimensions: the length of the prism L and the diameter $D = \frac{a\sqrt{3}}{2}$, where a is the side of a hexagonal cross section. Even in this most simple case one should deal with two-dimensional distribution functions $f(D,L)$. Note, that functions $f(D,L)$ can be approximately reduced to one-dimensional functions $f(D)$ due to the existence of empirical relationships between the length of crystals and their diameter in natural clouds (Auer and Veal, 1970). For instance, it holds approximately that $L/D = 4$ for long ($L > 2D$) solid columns.

It should be pointed out, however, that the whole conception of particle distribution functions is broken for ice clouds. Indeed, these functions are introduced for the sake of simplicity. For instance, it is often suffice to use only two numbers, the mode of the PSD and its dispersion for the characterization of statistical properties of water droplets distributions in warm clouds(Deirmendjian, 1969).

The crystalline clouds should be characterized at least by 80 multi-dimensional particle size distributions, if one would like to use the classification of crystals, developed by Magano and Lee(1966). There is not much use of such an approach, of course. Thus, there is a necessity of introduction of a new way for particle characterization of complex particulate systems such as ice clouds. The same problem arizes in the optics of mineral aerosol (Volten et al., 2001)

One of possible solutions of the problem lays in the characterization of ice crystals in an elementary volume of a cloudy medium by the function

$$f\left(\vec{a},\vec{b}\right) = \sum_{r=1}^{N} c_r f_r(\vec{a}) + \sum_{i=1}^{M} c_i f_i\left(\vec{b}\right), \tag{2.25}$$

where $f_r(\vec{a})$ is the size distributions of particles of a regular shape (e.g., hexagonal plates or columns), $f_i\left(\vec{b}\right)$ is the statistical distributions of particles with random surfaces or so-called irregularly shaped particles. Actually, functions $f_i\left(\vec{b}\right)$ present statistical distributions of some statistical characteristics of particles (e.g., average radii, correlation lengths, etc.). Values of c_i and c_r present concentrations of different crystal habits. Clearly, the simplest case is to consider the function $f\left(\vec{a},\vec{b}\right)$ as a sum of two functions:

$$f\left(\vec{a},\vec{b}\right) = c_1 f_1(\vec{a}) + c_2 f_2\left(\vec{b}\right), \tag{2.26}$$

where function $f_1(\vec{a})$ presents particles of a regular shape (say, hexagonal cylinders) and the function $f_2(\vec{b})$ presents statistical parameters of a single particle of irregular shape.

This irregularly shaped particle can be presented, e.g., as a fractal (Macke and Tzchiholz, 1992; Macke et al., 1996). It should be pointed out that the function $f_2(\vec{b})$ in this case represents "fictive" particles, which does not exist in a cloud at all. However, ice cloud optical characteristics, calculated using $f_2(\vec{b})$, indeed represent the optical characteristics of particles with extremely diverse shapes quite good(Macke et al., 1996).

Similar approach to the optical characterization of irregularly shaped particles was developed by Peltoniemi et al. (1989), Peltoniemi(1993) and Muinonen et al., 1996. It is based on Monte-Carlo calculations of light scattering by a large particle with a rough surface. The model of spheres with rough surfaces was successfully applied to the optical characterization of irregularly shaped aerosol particles (Volten et al., 2001). No doubt that this model the model can be extended for the case of crystalline clouds as well. For this one needs to change the index of refraction, which is somewhat lower for ice than for mineral fraction of the atmospheric aerosol. Also the parameters of the irregularity of a ''fictive'' particle should be changed accordingly. This is due to different morphology of ice crystals (Magano and Lee, 1966) as compared to mineral aerosol (Okada et al., 2001)

3. Local optical characteristics of cloudy media

3.1 Water clouds

3.1.1 Extinction coefficient

The information presented in the Section 2 can be used as an input for studies of light interaction with cloudy media on a global or local scale. In particular, the attenuation of a direct light beam with the intensity I_0 in a cloudy medium is governed by the following equation:

$$I = I_0 \exp(-\tau / \cos\vartheta_0), \tag{3.1}$$

where I_0 is the intensity of an incident light, I is the intensity of the transmitted direct light, ϑ_0 is the solar zenith angle and

$$\tau = \int_0^H \sigma_{ext}(z)dz \tag{3.2}$$

is the optical thickness of a cloud. Here H is the geometrical thickness of a cloud, σ_{ext} is the extinction coefficient. It is assumed that a cloud is contained inside horizontally infinite homogeneous plane-parallel layer, which is, of course, a great simplification of a real cloud field (Cahalan at el., 1994, 2001). The value $l = 1/\sigma_{ext}$ is called the photon free path length. It gives us the average distance between photon scattering events in the cloud. Knowing the average number of scattering events *n*, it is easy to find the average distance *L=nl,* which photon travels in the medium before escape. The average total time $T=L/c$ (c is the group speed of light in a cloud), which photon spends in a cloud, can be also easily estimated.

Note, that the value of l is often in the range 10 - 200 meters for water clouds. Thus, the value of σ_{ext} is in the range $0.005-0.1m^{-1}$, depending on the cloud type. It could be even larger for dense fogs. Note, that the value of σ_{ext} determines the meteorological range of visibility (Zege et al., 1991; Liou, 1992), which is defined as $S_m = \frac{3.91}{\sigma_{ext}}$. In particular, we have at $\sigma_{ext}=0.1\,m^{-1}$: $S_m = 39.1m$. In the cloudless atmosphere this number is approximately 1000 times larger. Thus, the small range of visibility in clouds and fogs effects, e.g., the air traffic in lower atmosphere. It also influences the vision and detection systems and makes the transportation on the ground level more difficult.

The more advanced understanding of the image reduction by cloudy media can be achieved on the base of the image transfer theory (Zege et al., 1991; Zege and Kokhanovsky, 1994, 1995; Barun, 1995, 2000; Zuev et al., 1997; Katsev et al., 1998). This theory treats a cloud as a high-frequency spatial filter. This filter reduces high spatial frequencies of observed objects in a great extent. It means, that fine features of objects are lost due to multiple light scattering process. It is interesting to note, that clouds with larger particles, have ‘’better’’ optical transfer functions(higher values) as compared to clouds with very fine droplets. This fact can be used for cloud microstructure monitoring purposes (Zege and Kokhanovsky, 1992). However, we will not go in any details here and refer to an excellent monograph by Zege et al(1991), where the subject is considered at a considerable length.

The extinction coefficient varies with the height in a cloud body (see Eq. (3.2)). It can be calculated from the following equation at a given value of the vertical coordinate z

$$\sigma_{ext} = N\int_0^\infty f(a)C_{ext}da\,, \tag{3.3}$$

where N is the number concentration of particles. The value of the extinction cross section C_{ext} is obtained on the base of the Mie theory, which is quite complicated (Shifrin, 1951; van de Hulst, 1957; Kerker, 1969; Bohren and Huffman, 1983). Fortunately, it was found that it holds approximately(Shifrin, 1951):

$$C_{ext} = \Sigma/2 \tag{3.4}$$

for water droplets in the visible range of the electromagnetic spectrum. Here Σ is the surface area of a droplet. One obtains from Eqs. (3.3), (3.4):

$$\sigma_{ext} = \frac{N\langle\Sigma\rangle}{2}\,. \tag{3.5}$$

Thus, the extinction coefficient depends on the product of the number concentration of particles and their average surface area. The value of N is related to the volume concentration of particles C_v by the following formula:

$$N = \frac{C_v}{\langle V \rangle}, \tag{3.6}$$

where $\langle V \rangle$ is the average volume of particles. Thus, it follows from Eqs. (3.5), (3.6):

$$\sigma_{ext} = \frac{3C_v}{2a_{ef}}, \tag{3.7}$$

where we introduced the effective radius

$$a_{ef} = \frac{3\langle V \rangle}{\langle \Sigma \rangle}. \tag{3.8}$$

One can see that the extinction coefficient decreases with the size of particles at $C_v = const$. The extinction coefficient can be also expressed via the liquid water content C_w, which is often measured in cloudy media:

$$\sigma_{ext} = \frac{3C_w}{2\rho a_{ef}}, \tag{3.9}$$

where $\rho = 10^6\, g/m^3$ is the density of water. It follows for typical values $a_{ef} = 6\mu m$ and $C_w = 0.4\, g/m^3$: $\sigma_{ext} = 0.1 m^{-1}$. This is rather dense cloud with the ratio $I_0 / I = e$ already at the distance 10m(see Eq. (3.1)).

We should underline two peculiarities of water clouds, which follow from Eq. (3.9). First of all the extinction does not depend on the wavelength λ and secondly it depends only on the ratio of liquid water content to the effective radius a_{ef}. The first point leads to the conclusion that clouds do not change the spectral composition of a direct light beam in visible. Thus, spectral transmittance methods of particulate media microstructure determination(Shifrin and Tonna, 1992) can not be applied in cloud optics. Another important property, which follows from Eq. (3.9), is independence of the value of σ_{ext} on the type of particle size distribution. Even the width of the PSD is of no importance at a given value of the effective radius of particles. This is the genuine reason why the satellite methods of cloud microstructure determination (Arking and Childs, 1985; Nakajima and King, 1990; Han et al., 1994) are concerned mostly with the retrievals of values a_{ef} and the liquid water path(see Eqs. (2.22), (3.9)):

$$w = \frac{2}{3}\rho a_{ef}\tau \qquad (3.10)$$

and not the droplet size distribution itself.

The type of the particle size distribution in clouds is not possible to find from extinction measurements in visible. On the other hand, this has an advantage. One can calculate the extinction coefficient of a cloudy media even with limited information on statistical properties of a particulate medium. This is especially important for ice clouds, as we will see later.

Simple Eq. (3.9) is valid only in the visible range of the electromagnetic spectrum. Calculations for larger wavelengths should be performed with the Mie theory. However, a simple correction of Eq. (3.9) allows for its use also in the near-infrared region of the electromagnetic spectrum(Kokhanovsky and Zege, 1995;1997a;1997b;Kokhanovsky, 2001a):

$$\sigma_{ext} = \frac{3C_w}{2\rho a_{ef}}\left\{1 + \frac{A}{\left(ka_{ef}\right)^{2/3}}\right\}, \qquad (3.11)$$

where $k = 2\pi/\lambda$ and A is parameter, which only slightly depends on the dispersion of the particle size distribution. It follows that for most typical widths of particle size distributions in clouds: $A = 1.1$(Kokhanovsky and Zege, 1997b). It follows from Eq. (3.11) that the extinction coefficient of cloudy media increases with the wavelength in the visible and near infrared. The spectral dependence of σ_{ext} for the gamma PSD (2.3) at $a_{ef} = 4\mu m$, μ=6 is presented in Fig. 3. The data were obtained with simple Eq. (3.11) and the Mie theory, assuming that $C_w = 0.1 g/m^3$. The value of A was equal to 1.1. One can see that the accuracy of Eq. (3.11) decreases with the wavelength/size ratio. However, the error is smaller than 5 % at $\lambda < 2\mu m$ and $a_{ef} > 4\mu m$. Smaller values of a_{ef} are of a rare occurrence in natural clouds(Han et al., 1994). Note, that simple Eq.(3.9) gives a constant value equal to 0.375 m^{-1} in the case, presented in Fig.3. We see that Eq.(3.11) increases the accuracy of calculations and provides us with correct wavelength dependent of the extinction coefficient (at least till the wavelength equal to 2 μm).

The accuracy of Eq. (3.11) at larger wavelengths can be increased if one accounts for the dependence of the extinction cross section on the refractive index of particles. It could be done, for instance, in the framework of the van de Hulst approximation (Ackerman and Stephens, 1987). Parametrizations, developed by Mitchell (2000), and Harrington and Olsson(2001) can be used even outside the geometrical optics limit.

Clearly, the accuracy of Eq. (3.11) increases with the value of a_{ef} . Thus, the case presented in Fig. 3 provides us with the maximal error of the approximation (3.11) for terrestrial clouds, having a_{ef} >4 μm .

3.1.2.Absorption coefficient

Clouds both scatter and absorb incident radiation. The probability of photon absorption β is defined by the following equation:

$$\beta = \frac{\sigma_{abs}}{\sigma_{ext}}, \qquad (3.12)$$

where σ_{abs} is the absorption coefficient and σ_{ext} is the extinction coefficient(see, e.g., Eq.(3.11)). The value of β is close to zero in visible and near infrared which allow to obtain simple equations for reflection and transmission functions of cloudy media in terms of series $\beta^{n/2}$, where n is an integer number (Rozenberg, 1962, 1967; van de Hulst, 1980, Minin, 1988). The absorption coefficient is found from the following equation:

$$\sigma_{abs} = N\int_0^\infty C_{abs} f(a)da, \qquad (3.13)$$

where C_{abs} is the absorption cross section. It follows for the scattering coefficient: $\sigma_{sca} = \sigma_{ext} - \sigma_{abs}$. The absorption cross section can be calculated from the Mie theory for spherical particles. Another way to calculate the absorption cross section, which is valid for nonspherical particles as well, is to use the integral(Kokhanovsky, 2001a, Markel, 2002):

$$C_{abs} = \frac{2k}{|\vec{E}_0|}\int_V n(\vec{r})\chi(\vec{r})\vec{E}(\vec{r})\vec{E}^*(\vec{r})d^3\vec{r}, \qquad (3.14)$$

where $\vec{E}_0$ is the electric field of an incident wave, $\vec{E}$ is the electric field inside a particle, $k = 2\pi/\lambda$ is the wave number, $m = n - i\chi$ is the refractive index, V is the volume of a particle. The first coarse approximation is to assume that the value of

$$\vec{E}(\vec{r})\vec{E}^*(\vec{r}) \approx B(n)E_0(\vec{r})\vec{E}_0^*(\vec{r}) \qquad (3.15)$$

due to the weakness of absorption. Thus, it follows in this case

$$C_{abs} = B(n)\alpha V, \qquad (3.16)$$

where $\alpha = \frac{4\pi\chi}{\lambda}$ and we assumed that a particle is uniform. Clearly, we have as $n \to 1: B \to 1$ for arbitrarily shaped particles. It follows from Eqs. (3.13), (3.16):

$$\sigma_{abs} = B(n)\alpha N\langle V\rangle \qquad (3.17)$$

or

$$\sigma_{abs} = B(n)\alpha C_v \quad . \qquad (3.18)$$

We have calculated the value $B = \sigma_{abs} / \alpha C_v$ for the PSD ((2.3) with $\mu = 6$ and $a_{ef} = 4\mu m$ with the Mie theory and presented it in Fig.4 as the function of the light wavelength. One can see that indeed in a full correspondence with approximate Eq. (3.18) the value of B almost does not depend on the imaginary part of the refractive index and the effective diffraction parameter $x_{ef} = ka_{ef}$ of water droplets till $\lambda \approx 1.8\,\mu m$. For larger values of λ Eq. (3.15) becomes invalid due to higher absorption of light by droplets or crystals. We have from Fig.4: $B \approx 5/3$.

The accuracy of approximation (3.18) is better than 5% till $\lambda =$ $1.8\mu m$ for values of a_{ef} in the range 4 – 6 μm . For larger particles the accuracy reduces due to the neccesity to account for terms proportional to α^2 and higher in Eq. (3.18) (Kokhanovsky and Macke, 1997; Kokhanovsky, 2001a).

Eq. (3.18) can be written as

$$\sigma_{abs} = \frac{B(n)\alpha C_w}{\rho}, \tag{3.19}$$

where C_w is the liquid water content and ρ is the density of water. One can see that the spectral variation of the absorption coefficient of cloudy media approximately coincides with the spectral dependence of the absorption coefficient of liquid water itself at least up to 1.8 μm. The same applies to ice crystals. This feature can be used for the cloud thermodynamic phase determination by remote sensing techniques.

It should be pointed out that water droplets can collect absorbing particles from surrounding air. Liquid aerosol particles can penetrate into droplets. They dissolve and change the value of α in Eq. (3.18). Soot particles can form a discontinuous layer on the surface of a droplet. All these effects will produce an enhancement of the absorption coefficient as compared to simple Eq. (3.18) (Prishivalko et al., 1984). Note, that effects of aerosol particles (e.g., soot) can be modelled by inserting into Eq. (3.18) the value of

$$\alpha = \alpha_W + \sum_{i=1}^{N} c_i \alpha_i, \tag{3.20}$$

where α_W and α_i are absorption coefficients of water and impurities respectively, c_i is the relative concentration of impurities. The problem of black carbon influence on the light absorption in cloud was studied in detail by Chylek et al.(1996).

The parameter of considerable importance in the radiative transfer problems is not the absorption coefficient itself but the ratio of the absorption and extinction coefficients. It follows from Eqs. (3.11), (3.18)

$$\beta = \tilde{B}\alpha a_{ef}, \tag{3.21}$$

where $\tilde{B} = \frac{2}{3} B(n)\left(1 + A x_{ef}^{-2/3}\right)^{-1}$. The value of β is obtained from the measurement of the reflection function of clouds in the infrared (Nakajima and King, 1990). Thus, Eq. (3.21) can be used to find the effective radius of droplets, which corresponds to a given value of β. The uncertainty in the value of α (see Eq. (3.20)) can introduce additional errors in the retrieved values of the effective radius of droplets :

$$a_{ef} = \frac{\beta}{B\alpha}. \tag{3.22}$$

For instance, it follows: $\frac{\delta\alpha_{ef}}{\alpha_{ef}} = -\frac{\delta\alpha}{\alpha}$. We see, therefore, that additional aerosol absorption makes the value of α larger and the effective radius a_{ef} (as obtained from Eq. (3.22)) smaller as compared to values, obtained from in-situ measurements(e.g., using laser diffractometers). This was found also experimentally(Nakajima et al., 1991). To correct for this effect, one should use the value of α (3.20) in Eq. (3.22). The problem is, however, complicated by the fact that values of absorption coefficients and concentrations of impurities are not known *a priori*. On the other hand, one can formulate the inverse problem in such a way that they can be derived from measured spectral reflectances (Zege et al., 1998) simaltaneously with the value of the effective radius of droplets.

Eq. (3.18) is limited to the case of weak absorption. It can be generalized to account for larger absorption of light by water droplets. We found, parametrizing the Mie theory results, that

$$\sigma_{abs} = B\alpha C_v\left[1 - \alpha a_{ef}\right]. \tag{3.23}$$

Eq. (3.23) transforms to (3.18) for small values of absorption.

In conclusion, we present the spectral dependence of the probability of photon absorption in Fig.5. Data were obtained using the Mie theory and approximate solutions, given by Eqs. (3.23), (3.12), (3.11). The effective radius of particles was equal to 4 and 16 micrometers. The accuracy of the approximation is better than 10% at wavelengths smaller than 2 micrometers(see Fig.6).

3.1.4. Phase function

Until now we have considered only the extinction and absorption characteristics of cloudy media. However, clouds not only attenuate propagated sygnals and absorb light. They also scatter incident light in all directions. Generally speaking, the probability of photon scattering by a droplet depends on the size of the droplet and the energy of the incident photon(or the light wavelength).

Now we should characterize the probability of photon scattering in a given direction, specified by the scattering angle θ (θ=0 corresponds to the forward scattering). It is known , that the intensity of light scattered by water droplets is much larger in the forward hemisphere than it is in the backward directions(Deirmendjian, 1969; Kokhanovsky et al., 1998a).

Let us introduce the probability dP of light scattering in the direction, specified by the vector $\vec{\Omega}$ inside the solid angle $d\Omega$. Clearly, this probability will be proportional to the value of $\frac{d\Omega}{4\pi}$. Namely, we have:

$$dP = x(\Omega)\frac{d\Omega}{4\pi}, \tag{3.24}$$

where $x(\Omega)$ is the coefficient of proportionality. It follows from Eq. (3.24):

$$\int_{4\pi} x(\Omega)\frac{d\Omega}{4\pi} = P. \tag{3.25}$$

The value of P represents the probability of photon survival in the scattering process. It is equal to 1 if there is no absorption of light by a particle and it is smaller then 1 if some photons are absorbed by a particle. Clearly, the probability of photon survival P is equal to the single scattering albedo(Chandrasekhar, 1950):

$$w_0 = \frac{\sigma_{sca}}{\sigma_{ext}}, \tag{3.26}$$

where $\sigma_{sca} = \sigma_{ext} - \sigma_{abs}$. It should be pointed out that $\beta = \sigma_{abs}/\sigma_{ext}$ is a small parameter for water clouds (see Fig.5) in visible and $P \equiv w_0 = 1 - \beta \approx 1$ in this case. The value $p(\Omega) = x(\Omega)/P$ is called the phase function. It is equal to one if the probability of light scattering does not depend on the angle(see Eq. (3.25)). Note, that the value of $x(\Omega)$ is sometimes also called the phase function or the scattering indicatrix(Ishimaru, 1978; van de Hulst, 1980). Thus, one should be careful with normalization factors.

The light scattering by water droplets is azimuthally symmetric due to their spherical shape. Thus, one can simplify Eq. (3.25) and obtain after integration with respect to the azimuth angle:

$$\frac{1}{2}\int_0^{\pi} p(\theta)\sin\theta d\theta = 1. \tag{3.27}$$

The phase function (or the scattering indicatrix) can be calculated from the Mie theory(Shifrin, 1951). It does not depend on the concentration of particles by the definition. It depends, however, on their refractive index and size. Functions $p(\theta)$, obtained from the Mie theory, for different particle size distributions are presented in Fig. 7 at $\lambda = 0.65\mu m$. Main features of the phase functions of water clouds in visible are:

- sharp forward-backward asymmetry ;

- weak dependence on the size of droplets in the region of scattering angles from 20

$,\theta$

till 60 degrees;
- enhanced scattering near rainbow θ_r (approximately 138 degrees for water droplets) and glory θ_g (180 degrees) scattering angles (Tricker, 1970; Greenler, 1980; Konnen, 1985; Spinhirne and Nakajima, 1994);
- strong forward peak.

Amplitudes of peaks at angles $\theta = 0, \theta_r, \pi$ can be used as indicators of the size of droplets (van de Hulst, 1957; Shifrin and Tonna, 1992).

Approximate equations for the phase functions of water clouds were presented by many authors (e.g., see Shifrin(1951), van de Hulst(1957), Nussenzweig(1992), Zege et al.(1993); Kokhanovsky and Zege(1997b); Grandy(2000)). We found, parametrizing the Mie theory results, that the phase function of a cloud can be presented by the following simple equation:

$$p(\theta) = Q\exp(-s\theta) + \sum_{i=1}^{5} b_i \exp(-\beta_i(\theta-\theta_i)^2),$$

where *Q=17.7, s=3.9,* θ is given in radians and constants β_i, θ_i are given in Table 3. The shortcoming of this equation is that it does not account for the influence of the size of droplets on the phase function. However, it can be useful if one is interested not in precise numbers but in general understanding of multiple light scattering in cloudy media (Zege et al., 1993, 1995; Katsev et al., 1998).

The solution of the radiative transfer equation, which describes the radiative transfer through cloudy media, is simplified if one uses the expansion of the phase function in series:

$$p(\theta) = \sum_{s=0}^{\infty} a_s P_s(\cos\theta) . \qquad (3.28)$$

Here $P_s(\cos\theta)$ is the Legendre polynomial. Note, however, that in practice the number of coefficients a_s could be taken to be finite. It is approximately equal to *2kd*, where *d* is the average diameter of particles and *k* is the wavenumber (Kokhanovsky, 1997). The coefficients a_s for cloudy media calculated with the Mie theory at $\lambda = 0.65\mu m$ for the PSD (2.3) with $\mu = 6$ are presented in Fig.8. Approximate results for them, based on the combination of the geometrical optics and the Fraunhofer diffraction, were obtained by Kokhanovsky(1997, 2001a). They can be used to avoid long numerical calculations with the Mie theory as $x_{ef} \to \infty$.

We would like to underline, that both geometrical optics results (Kokhanovsky, 1997) and data, presented in Fig.8, show that the number $s^{\max}$, which corresponds to the largest coefficient $a_s^{\max}$, is equal approximately to $4x_{ef}/5$. Also we have approximately: $s^{\max} = a_s^{\max}/2$.

It follows from Eq. (3.28), using the orthogonality of Legendre polynomials:

$$a_s = \frac{2s+1}{2}\int_0^{\pi} p(\theta) P_s(\cos\theta)\sin\theta d\theta \ . \tag{3.29}$$

In particular, we have: a_0 =1 (see Eq. (3.27)) and $a_1 = 3g$, where

$$g = \frac{1}{2}\int_0^{\pi} p(\theta)\sin\theta\cos\theta d\theta \tag{3.30}$$

is the so-called asymmetry parameter. It gives the average cosine of the photon scattering angle inside the cloud. Note, that the value of *g* determines(together with the extinction coefficient) the coefficient of photon diffusion $D = \left[3\sigma_{ext}(1-g)\right]^{-1}$ in a cloudy medium.

We stress that it is the value of integral (3.30) (and not the phase function itself), which determines a number of important radiative characteristics of clouds (Zege et al., 1991). For instance, we have for the value of the total cloud reflectance or the spherical albedo (Kokhanovsky, 2001a): $r = \exp(-4\sqrt{\beta/3(1-g)})$. Here we assumed that the cloud is semi-infinite. Thus, the asymmetry parameter needs a special attention. We will consider this parameter in next Section in more detail.

3.1.4 The asymmetry parameter

Let us represent the phase function $p(\theta)$ as

$$p(\theta) = \frac{\sigma_{sca}^D p^D(\theta) + \sigma_{sca}^G p^G(\theta)}{\sigma_{sca}^D + \sigma_{sca}^G}. \tag{3.31}$$

Here $p^D(\theta)$ is the phase function associated with the Fraunhofer diffraction of light by a droplet, $p^G(\theta)$ is the phase function, associated with rays, which penetrate the particle, reflect and refract inside the droplet, σ_{sca}^G and σ_{sca}^d are scattering coefficients, associated with these processes. Note, that Eq. (3.31) corresponds to the van de Hulst's(1957) localization principle. It holds only approximately and ignores the possible interference of waves (Glautshing and Chen, 1981) , which originate due to separate diffraction and geometrical optics scattering processes.

It follows from (3.30), (3.31):

$$g = \frac{\sigma_{sca}^D g^D + \sigma_{sca}^G g^G}{\sigma_{sca}^D + \sigma_{sca}^G}, \tag{3.32}$$

where

$$g^D = \frac{1}{2}\int_0^{\pi} p^D(\theta)\sin\theta\cos\theta d\theta, \tag{3.33}$$

$$g^G = \frac{1}{2}\int_0^{\pi} p^G(\theta)\sin\theta\cos\theta d\theta. \tag{3.34}$$

One can obtain (van de Hulst, 1957; Kokhanovsky, 2001) that $g^D \approx 1$. Note, that the function $p^G(\theta)$ does not depend on the size of large particles in visible(van de Hulst, 1957; Kokhanovsky and Nakajima, 1998). Thus, g^G depends on the refractive index *n* of water only in this case. The value of *n* does not change considerably in the visible region of the electromagnetic spectrum. We will assume that n = 1.333. Then it follows: $g^G = 0.7686$ (Kokhanovsky, 2001a) and g = 0.8843 according to Eq. (3.32), where we accounted for the equality $\sigma^D_{sca} = \sigma^G_{sca}$, which also holds in the visible region of the electromagnetic spectrum(van de Hulst, 1957). The value of asymmetry parameter g of water clouds on practice weakly depends on the size of particles and wavelength. This dependence can be approximated by the following equation(Kokhanovsky, 2001a):

$$g = g_0 - \frac{C}{x_{ef}^{2/3}}, \tag{3.35}$$

where C is the constant, which does not depend on the size of droplets(but possibly depends on their shape). Results of calculations of the value $C = (g_0 - g)\, x^{2/3}$ with the Mie theory are presented in Fig.9 for the particle size distribution (2.3) at $\mu = 6$, $a_{ef} = 4\mu m$ and $\lambda \le 1\mu m$. One can see that $C \approx \frac{1}{2}$. Thus, we have from Eq. (3.35) for the value of $G = 1 - g$, which is often called the co-asymmetry parameter:

$$G = 0.12 + \frac{1}{2x_{ef}^{2/3}}. \tag{3.36}$$

This is the value of G and not g itself, which plays an important role in the radiative transport in cloudy media. For instance, values of g equal to 0.7 and 0.8 are comparatively close to each other. However, correspondent values of G (0.3 and 0.2 respectively) differ considerably. One should remember this fact, while comparing cloudy media with different values of the asymmetry parameter.

It is important to understand the physical meaning of the parameter G. This meaning can be established using the expansion of $\cos\theta = 1 - \frac{\theta^2}{2} + \ldots$ in Eq. (3.30). Then it follows: $G = \frac{\langle \theta^2 \rangle}{2}$, where $\langle \theta^2 \rangle = \frac{1}{2}\int_0^{\pi} \theta^2 p(\theta) \sin\theta d\theta$ is the averaged square of the scattering angle. Note, that we neglected high order terms in the expansion of $\cos\theta$, which is possible due to a peaked character of the phase function in Eq.(3.30).

Thus, we see that it is the average value of the squared scattering angle, which is responsible for the radiative transport in cloudy media. In particular, semi-infinite clouds with similar values of ratios $\beta / \langle \theta^2 \rangle$ have also close values of the total reflectance. The smaller values of $\langle \theta^2 \rangle$ means that photons penetrate to larger depths in cloudy media before their escape back to the free space, where the source of radiation is located. This also makes the total path lengths of photons in cloudy media larger, which in turn produces the larger total absorption of radiation inside clouds with larger values of $\langle \theta^2 \rangle$.

The total fraction of radiation, absorbed by a semi-infinite cloud is given by the following expression (Kokhanovsky, 2001a):

$$A = 1 - \exp\left\{-4\sqrt{\frac{\beta}{3(1-g)}}\right\}$$

or

$$A = 4\sqrt{\frac{2\beta}{3\langle\theta^2\rangle}}$$

as $\beta \to 0$. Clearly, the ratio

$$n = \frac{A}{\beta} \approx \frac{\nu}{\sqrt{\beta\langle\theta^2\rangle}},$$

where $\nu = 4\sqrt{2/3}$ can be used as an estimation of the average number of photon scattering events in a cloud layer. It is inversely proportional to $\sqrt{\beta\langle\theta^2\rangle}$. So smaller values of β and $\langle\theta^2\rangle$ give us larger values of n as one might expect.

Eq.(3.36) can be used only for nonabsorbing channels (e.g., in visible). However, it can be modified in a full analogy with Eq. (3.23) to account for light absorption at $\lambda \geq 1\mu m$ (Kokhanovsky, 2001a):

$$G = 0.12 + \frac{1}{2x_{ef}^{2/3}} - 0.15\alpha a_{ef}\,. \qquad (3.37)$$

Comparisons of calculations with simple Eq. (3.37) and the Mie theory are presented in Fig.10. They were performed for the particle size distribution (2.3) at $\mu = 6$, $a_{ef} = 4\mu m$ and $a_{ef} = 6\mu m$ for wavelengths $\lambda \leq 2.4\mu m$. It follows that the accuracy of Eq.(3.37) is better than 5% at $a_{ef} = 6\mu m$ in visible and near infrared till $\lambda \leq 2.3\mu m$. It is also better than 5% for droplets with $a_{ef} = 4\mu m$, but for values of $\lambda \leq 2.0\mu m$.

Another important characteristic of the phase function of a cloudy medium is the probability of light scattering in the backward hemisphere:

$$F = \frac{1}{2}\int_{\pi/2}^{\pi} p(\theta)\sin\theta\cos\theta d\theta\,.$$

It was shown (Kokhanovsky et al., 1998) that it holds approximately for water clouds:

$$F = 0.03 + \frac{1}{5x_{ef}^{2/3}}\,.$$

It suggests that approximately 97% of light is scattered by a local volume of a cloudy medium in the range of angles smaller than 90 degrees. The backscattering signal is, therefore, quite low. It increases, however, due to multiple scattering processes, which take place inside a cloud.

3.2. Ice clouds

3.2.1. Extinction coefficient

Local optical characteristics of ice and mixed clouds can not be calculated so easy as it could be done for water clouds. In particular, one can not rely on the Mie theory (Shifrin, 1951) anymore. This is related to the complex shape and internal structure of ice crystals (Takano and Liou, 1989; Macke, 1993, 1994; Macke et al., 1996, 1998; Mitchell and Arnott, 1994; Yang and Liou, 1998, 2000, 2001). Main results obtained in the optics of crystalline media were summarized by Volkovitsky et al. (1984) and Liou (1992).

The size of ice crystals is usually much larger than the wavelength of the incident radiation. Thus, the extinction cross section C_{ext} does not depend on the wavelength and the refractive index of particles(Shifrin, 1951; van de Hulst, 1957):

$$C_{ext} = 2s\,, \tag{3.38}$$

where s is the cross-section of a particle, projected on the plane perpendicular to the incidence direction. It follows for N identical crystals in a fixed orientation:

$$\sigma_{ext} = 2Ns\,, \tag{3.39}$$

where N is number concentration of particles and σ_{ext} is the extinction coefficient. However, identical crystals do not exist in ice clouds. They differ by their shape, size and orientation. The extinction coefficient can be calculated as

$$\sigma_{ext} = 2N\bar{s}\,, \tag{3.40}$$

where $\bar{s}$ is the average cross section of particles. This equation can be written in the following form:

$$\sigma_{ext} = 2C_v \frac{\bar{s}}{\bar{V}}\,, \tag{3.41}$$

where C_v is the volumetric concentration of crystals and $\bar{V}$ is the average volume of crystals ($C_v = N\bar{V}$). It follows for convex crystals of the same shape in random orientation(van de Hulst, 1957): $\bar{\Sigma} = 4\bar{s}$, where $\bar{\Sigma}$ is the average surface area of particles. Thus, one can obtain:

$$\sigma_{ext} = \frac{3C_v}{2a_{ef}}\,, \tag{3.42}$$

where $a_{ef} = \frac{3\bar{V}}{\bar{\Sigma}}$ is the effective radius of particles. This result is similar to Eq. (3.7). Only a portion of ice crystals (e.g., hexagonal plates and columns) are convex. For this and other reasons one should expect that Eq. (3.42) will provide us with only a coarse approximation. Let us introduce the equivalent size of crystals:

$$a_e = \frac{3C_v}{2\sigma_{ext}}. \qquad (3.43)$$

Then it follows instead of Eq. (3.42):

$$\sigma_{ext} = \frac{3C_v}{2a_e}. \qquad (3.44)$$

Crystal media with particles of different shape, orientation and size can be characterized in this case just by one number, namely the equivalent size a_e. This size is equal to the effective radius for spherical polydispersions or convex crystals.

3.2.2 Absorption coefficient

The absorption cross section of a single crystal can be found in a full analogy with Eq.(3.16):

$$C_{abs} = B\alpha V, \qquad (3.45)$$

where V is the volume of a crystal, $\alpha = 4\pi\chi/\lambda$ and B is the coefficient proportionality, which depends on the shape and real part of the refractive index of particles, but not on their size.This follows from Eq. (3.14).

It follows for the ensemble of crystals of identical shapes:

$$\sigma_{abs} = B\alpha C_v \ . \qquad (3.46)$$

Let us suppose now that we have *N* distinct shapes of crystals in a cloud. Clearly, it follows instead of Eq. (3.46) in this case:

$$\sigma_{abs} = \alpha\sum_{j=1}^{N} B_j C_{v\,j}. \qquad (3.47)$$

The values of B_j do not depend on the imaginary part of the refractive index and the size of crystals. They depend only on the real part of the refractive index *n* of crystals and their shapes. Thus, the most probable values of $B = \frac{1}{C_v}\sum_{j=1}^{N} B_j C_{v\,j}$. in crystalline clouds can be found from experimental measurements of ratios $\sigma_{abs}/C_v\alpha$ in ice clouds of different microstructures.

The probability of photon absorption $\beta = \sigma_{abs}/\sigma_{ext}$ can be found from Eqs. (3.44), (3.47). Namely, it follows:

$$\beta = \Xi\alpha a_e, \qquad (3.48)$$

where

$$\Xi = \frac{2}{3C_v}\sum_{j=1}^{N} B_j C_{vj} \qquad (3.49)$$

It is readily apparent from this equation that the single scattering albedo

$$w_0 = 1 - \Xi\alpha a_e \qquad (3.50)$$

for crystalline media. The generalization of these equations on the case of arbitrary light absorption by droplets was given by Kokhanovsky and Macke(1998).

3.2.3 Phase function

The phase function of ice clouds in visible is again the average on the ensemble of phase functions of crystals with different shapes. It depends on the size of crystals in the small-angle scattering region. Mostly shape and structure of crystals is important at larger scattering angles. The phase function of hexagonal crystals outside the diffraction region is presented in Fig.12. This phase function was obtained using the Monte-Carlo ray tracing approach (Macke, 1994) for hexagonal ice cylindres in random orientation at the wavelength 0.5 μm. All cylinders were assumed to be identical and having the length 0.5 mm. The side of the cross-section was taken to be equal to 0.08 mm. One can see that the main feature of phase functions of this type are halos near the scattering angle 22 and 46 degrees. Greenler (1980) states that he observes 22° halos in Wisconsin on 70 to 80 days of a typical year. So these halo often appear in natural conditions. The second halo at 46° is of a rare occurrence in natural conditions due to the presence of crystals of other shapes, which wash out halo phenomena. Both rainbow and glory (Tricker, 1970; Greenler, 1980; Konnen, 1985) are absent for ice clouds.

These features can be used to find the thermodynamic phase of clouds from space on a global scale. The knowledge on the thermodynamic phase of clouds is of practical importance for climate studies. This is due to the fact that warm and ice clouds behave differently in respect to both solar and terrestrial radiation(Liou, 1992).

More realistic phase function of a crystalline cloud, which accounts also for the diffraction of light at small angles and size/shape distribution of crystals is tabulated by Takano and Liou(1989) and Liou(1992). It is characterized by the value of the asymmetry parameter $g=0.75$.

One of possibilities to avoid the calculation of phase functions of crystals of different shapes is to introduce the single “fictive” particle with the phase function, which is similar to the phase function of an ensemble. Clearly, the statistical properties of the surface of this particle should be somehow related to the statistical properties of an ensemble of scatterers. Particles with random stochastic surfaces were studied by Peltoniemi (1993), Macke(1994) and Muinonen et al., 1996. Depending on the parameters of roughness, they can quite well describe the phase functions of ice crystals(Macke, 1994) and mineral fraction of the atmospheric aerosol(Volten et al., 2001).

Th phase functions of a “fractal” particle is presented in Fig.13(Macke et al., 1996). It is characterized almost the same value of the asymmetry parameter as the phase function, presented in Fig.12$\left(g \approx 0.74\right)$. However, it is better suited to the description of complex systems such as crystalline clouds. In practice, one can use linear combinations of functions presented in Figs. 7, 12, 13. The weights of different contributions depend on the concentration of spherical particles, hexagonal cylinders and irregularly shaped particles in the cloud. Note, that other regular shapes such as plates can also contribute to the total phase function of an elementary volume of a cloudy medium.

Another complication is related to the horizontal orientation of crystals, which was found to be the case at least in 40% of ice cloud pixels studied, using satellite data, by Chepfer et al.(1999). The radiative transfer in ice clouds with horizontally oriented crystals was studied in detail by Liou(1992).

3.2.4 Asymmetry parameter

Asymmetry parameters of phase functions of ice crystals in visible depend on their shape, but not on the size of crystals (Macke et al., 1996; Kokhanovsky and Nakajima, 1998). This is due to large size of ice crystals in comparison with droplets. So wave corrections, such as given by the second term in Eq. (3.36), can be neglected. The real part of the refractive index of ice crystals is also of importance. However, it varies only slightly in the visible and this dependence can be neglected. Thus, the asymmetry parameter of the phase function of ice clouds will be the average value for an ensemble of particles of different shapes. Macke et al. (1998) found from extensive numerical calculations that the asymmetry parameter g is in the range 0.79 – 0.85 for columns, 0.83-0.94 for plates and 0.74 for polycrystals, represented by a ''fictive'' fractal particle, in visible. It is in the range 0.83-0.87 for spherical droplets. We see, therefore, that the value of g for cloudy media with ice crystals should be somewhere between 0.74 and 0.87 . The larger values of g mean that ice clouds are composed of plates only, which is never the case in natural conditions. We see that the co-asymmetry parameter $G=1-g$ changes by 100% from 0.13 for water clouds till 0.26 for irregularly shaped particles. These numbers are given only for the general orientation. What is the actual value of g for a given cloud can be found only from direct measurements in natural ice clouds. Such measurements were performed, e.g., by Garett et al.(2001), who found that the value of g is in the range 0.73-0.76, depending on the cloud area under investigation. Values of g were obtained from measurements of the phase function inside the ice cloud in the range of angles 10-175 degrees. In one case the value of g was appeared to be equal to 0.81. However, the crystals were evaporating in that area. So it can be considered as untypical case. We see that the asymmetry parameter of ice clouds does not vary considerably in visible and in average it is equal to 0.745, the value, which is close to that for a ''fractal'' fictive particle. It is also close to the value of g hexagonal crystals, which give the phase function, presented in Fig.12.

It is our believe, that the asymmetry parameter of crystal clouds should be taken as equal to 0.74 for the theoretical modeling of light propagation in crystal clouds. The correspondent value of G is equal to 0.26. This number does not depend on the size of crystals due to their large size as compared to the wavelength. It does not depend on the shape of crystals, because for a given cloud we have a statistical and very broad distribution of shapes, which produce in the end a saturated value of g for a completely chaotic scattering. This scattering can be modeled by a single fractal ''fictive'' particle. It should be stressed, however, that the calculation of g involves the averaging on the scattering angle(see Eq. (3.30)). Thus, the model of a chaotic scattering is more appropriate for the value of g than for the phase function itself. The phase function should be modeled as a combination of light scattering by regular and irregularly shaped particles with different weights as discussed above. Again such a model can be established only from measurements, performed in natural clouds.

Now we consider briefly the case of mixed ice – water clouds. Then we have for the value of G:

$$G = G_i + c_W G_W \,, \qquad (3.51)$$

where G_i and G_W are co-asymmetry parameters of ice and water particles respectively, c_W is the fraction of water droplets in a mixed cloud, defined as the ratio of number of droplets to a total number of particles in a mixed cloud. We see that one can obtain the value of c_W from measurements of G. This is an important parameter not only for cloud , but also for Earth climate research. The variability of G_W in mixed clouds is not very large. So, in the first approximation, we can assume the constant value of G_W. Measurements of Garett et al. (2001) suggest that $G_i = 0.26$ and $G_W = 0.13$. Then we have from Eq. (3.51):

$$c_W = \frac{G}{0.13} - 2 \qquad (3.52)$$

or approximately:

$$c_W = \frac{\langle \theta^2 \rangle}{4} - 2 \quad . \qquad (3.53)$$

The correlation between values of G and c_W were confirmed experimentally with the square of the correlation coefficient equal to 0.52 (Garett et al., 2001).

It is evident (Kokhanovsky and Macke, 1998; Kokhanovsky, 2001a) that in the near-infrared range of the electromagnetic spectrum the size of crystals generally influences the asymmetry parameter in such a way that the asymmetry parameter becomes larger. Detailed calculations of optical characteristics of ice crystals in infrared for different shapes of crystals were performed by Zhang and Xu(1995) and Macke et al.(1998). Different parametrization schemes for local optical characteristics of ice clouds were developed by Ebert and Curry(1992), Fu and Liou(1993), Fu(1998), Kokhanovsky and Macke(1998), Yang et al.(2000), Harrington and Olsson(2001).

4. Global optical characteristics of cloudy media

4.1 The visible range

Let us consider now the global optical characteristics of clouds, such as their reflection and transmission functions(van de Hulst, 1980). They can be measured remotely by airborne, satellite and ground-based radiometers and spectrometers(Kondratyev and Bunenko, 1984). The task of this Section is to introduce simple formulae, which can be used for the cloud global optical characteristics calculations. We will start from the reflection function of a cloudy medium under an assumption that the cloud can be represented as a homogeneous plane-parallel layer. The absorption of light by particles is neglected.

The reflection function of a cloud $R(\vartheta_0, \vartheta, \varphi)$ is defined as the ratio of reflected light intensity $I(\vartheta_0, \vartheta, \varphi)$ for the case of a cloud to that of an ideal Lambertian white reflector

$$R(\vartheta_0,\vartheta,\varphi)=\frac{I(\vartheta_0,\vartheta,\varphi)}{I^*(\vartheta_0)}, \tag{4.1}$$

where

$$I^*(\vartheta_0)=F\cos\vartheta_0 \tag{4.2}$$

is the intensity of light reflected from the ideally white Lambertian reflector, πF is the solar flux on the area perpendicular to the direction of incidence, ϑ_0 is the solar angle, ϑ is the observation angle and φ is the relative azimuth between solar and observation directions. Also $\mu=\cos\vartheta, \mu_0=\cos\vartheta_0$. It follows for the Lambertian ideally white reflector from Eq. (4.1): $R\equiv 1$. This result does not depend on the viewing geometry by definition.

Although clouds are white when looking from space, their reflection function $R(\vartheta_0,\vartheta,\varphi)$ is not equal to one. It depends on the viewing geometry. The results of calculations of the reflection function of an idealized semi – infinite nonabsorbing water cloud $R_\infty^0(\vartheta_0,\vartheta,\varphi)$ at the wavelength λ = *650 nm* and the nadir observation are presented in Fig.13. Calculations were performed for the gamma particle size distribution (2.3) at $\mu=6$. The values of the effective radius in Fig. 13 were 6 and 16 micrometers. This covers the typical range of variability of the effective radius in natural water clouds. We see that the function $R(\vartheta_0,\vartheta,\varphi)$ can be smaller and larger than 1 depending on the incidence angle. This implies that for particular viewing geometries cloud is even more reflective than the ideally white Lambertian surface. This is mostly due to peculiarities of the phase function of cloudy media (e.g., in the backscattering $(\vartheta\approx\vartheta_0,\ \varphi\approx\pi)$ region) for comparatively thick clouds. It follows from Fig.14 that in the range of solar angles 30 – 60 degrees and nadir observation the reflection function of a water cloud is almost equal to the reflection function of an ideally white Lambertian reflector. It differs from 1 not more than by 10% for these geometries.

The reflection function $R_\infty^0(\vartheta_0,\vartheta,\varphi)$ can be represented by the following simple approximate equation (Kokhanovsky, 2002):

$$R_\infty^0(\vartheta_0,\vartheta,\varphi)=\frac{b_1+b_2\cos\vartheta\cos\vartheta_0+p(\theta)}{4(\cos\vartheta+\cos\vartheta_0)} \tag{4.5}$$

where $\theta=\arccos(-\cos\vartheta\cos\vartheta_0+\sin\vartheta\sin\vartheta_0\cos\varphi)$ is the scattering angle, $p(\theta)$ is the phase function of a cloudy medium, b_1 and b_2 are constants. Eq. (5) obeys to the reciprocity principle (Zege et al., 1991). We have for nadir observations (Kokhanovsky, 2002): $b_1=1.48, b_2=7.76$. The comparison of approximate and exact data in Fig. (13) shows that the accuracy of Eq. (4.5) is better than 2% at $\vartheta_0<85^\circ$. Constants b_1 and b_2 for other viewing geometries can be found using parametrizations of results obtained from the exact radiative transfer codes (see, e.g., Mishchenko et al., 1999). Approximations for the function $R_\infty^0(\vartheta_0,\vartheta,\varphi)$ were also obtained by Melnikova et al.(2000).

Eq. (4.5) can be also used to find the reflection function of a finite cloud $R(\mu,\mu_0,\varphi,\tau)$ via the following equation (Germogenova, 1963; van de Hulst, 1980, Minin, 1988):

$$R(\mu,\mu_0,\varphi,\tau) = R_\infty^0(\mu,\mu_0,\varphi) - t(\tau)K_0(\mu)K_0(\mu_0)\,, \tag{4.6}$$

where

$$t = \frac{1}{0.75\tau(1-g)+\alpha} \tag{4.7}$$

is the global transmittance of a cloud, $K_0(\mu)$ is the escape function. The escape function is defined via the solution of the characteristic integral equation (van de Hulst, 1980). Note that parameters α and g in Eq. (4.7) are defined as follows(Sobolev, 1972):

$$\alpha = 3\int_0^1 K_0(\mu)\mu^2 d\mu\,, \tag{4.8}$$

$$g = \frac{1}{2}\int_0^\pi p(\theta)\sin\theta\cos\theta d\theta\,. \tag{4.9}$$

It should be pointed out that the escape function $K_0(\mu)$ only weakly depends on the cloud microstructure and can be presented by the following simple equation (Zege et al., 1991; Kokhanovsky, 2001a):

$$K_0(\mu) = \frac{3}{7}(1+2\mu)\,. \tag{4.10}$$

The function $K_0(\mu)$ calculated with exact radiative transfer code for g equal to *0.75, 0.85,* and *0.9* in the case of Heney-Greenstein phase function is presented in Fig.14. We see that $K_0(\mu)$ almost does not depend on g at $\mu \geq 0.2\,(\vartheta < 78^\circ)$. This is the case even for *g=0* and for the Mie-type phase functions. At the range of observation angles $\vartheta = 80^\circ - 90^\circ$ there is some dependence of the escape function on the microstructure of the cloud. However, the cloud top nonhomogenity plays a role at such grazing observation angles. So the problem can not be solved in the plane-parallel layer approximation in this case anyway.

The variability of $K_0(\mu)$ at $\mu = 0.2 - 1.0$ for different values of the average cosine of the scattering angle *g=0.75 – 0.9* is well inside 2% sensitivity corridor. This coincides with the error of Eq. (4.10) at $\mu \geq 0.2$ is smaller than 2%. Our discussion confirms the wide range of applicability of Eq. (4.10) in cloud optics. Note, that function (4.10) also describes the angular distribution of solar light transmitted by a cloud(Kokhanovsky, 2001a).

The substitution of Eq.(4.10) into Eq. (4.8) yields:

$$\alpha = \frac{15}{14} \approx 1.07 \tag{4.11}$$

independent of cloud microstructure. It should be pointed out that the value of α, numerically calculated by King (1987), assuming the fair weather cumulus cloud model, is given approximately by $\alpha = 1.07$, i.e., in agreement with our estimation. King(1987) used the Mie theory to find the phase function of a cloudy medium. Yanovitskij (1997) found the same value of α for Heney-Greenstein phase functions with asymmetry parameters in the range 0.0-0.9. This supports the approximation of using a fixed value of α in Eq. (4.7) , given by (4.11), independent of a cloud microstructure.

The accuracy of Eq. (4.6) is illustrated in Figs.15a,b. The error is less than 3% at $\tau \geq 5$ and $\lambda = 0.65\mu m$. This range of optical thicknesses is that most frequently observed in water clouds both with satellite and ground-based techniques (see Fig. 16, prepared from data given by Trishchenko et al.(2001). The small almost constant error at $\tau \geq 30$ is mostly due to the error of approximation (4.5) for a semi-infinite cloud. Errors are negligibly small for all practical purposes for optically thick cloud fields.

Eq. (4.6) is readily can be easily modified to account for the Lambertian light reflection from the underlying surface (Sobolev,1972):

$$\hat{R}(\mu,\mu_0,\varphi,\tau) = R(\mu,\mu_0,\varphi,\tau) + \frac{AT(\mu)T(\mu_0)}{1-Ar}, \qquad (4.12)$$

where $\hat{R}$ is the reflection function of a Lambertian surface-cloud system, A is the spherical albedo of the Lambertian surface, which may depend on the wavelength, $R \equiv \hat{R}(A=0)$,

$$T(\mu) = tK_0(\mu) \qquad (4.13)$$

is the diffuse transmittance of a cloud layer (Sobolev, 1972) and r is the spherical albedo of a cloud. Due to the energy conservation law we have that $r = 1 - t$ in visible, where we neglect small light absorption in a cloud body. Note, that we have neglected the direct solar light term in Eq. (4.12). This is possible due to a large thickness of clouds under consideration.

Finally, substituting Eq. (4.6) and Eq. (4.13) into Eq. (4.12) we have for the reflection function of a Lambertian surface-cloud system: :

$$\hat{R}\,(\mu,\mu_0,\varphi,\tau) = R_\infty^0(\mu,\mu_0,\varphi) - \frac{t\,(1-A)}{1-A\,(1-t)} K_0(\mu) K_0(\mu_0). \qquad (4.14)$$

This formula can be used as a basis for the semi-analytical cloud retrieval algorithm . Note, that $\hat{R}\,(\mu,\mu_0,\varphi,\tau) \equiv R_\infty^0(\mu,\mu_0,\varphi)$ at $A=1$.

4.2 The near-infrared range.

Unfortunately, relatively simple Eq. (4.6) can not be applied to the calculation of the reflection function of a finite cloud in the near-infrared region of the electromagnetic spectrum because of due to the presence of absorption bands of liquid water. Alternatively, the following formula applies (Germogenova, 1963; van de Hulst, 1980; King, 1987; Nakajima and King, 1992):

$$R(\mu,\mu_0,\varphi,\tau) = R_\infty(\mu,\mu_0,\varphi) - \frac{mle^{-2\gamma\tau}}{1-l^2e^{-2\gamma\tau}} K(\mu) K(\mu_0), \qquad (4.15)$$

where γ is the diffusion exponent, $K(\mu)$ and R_∞ are the escape function and the reflection function of an absorbing semi-infinite medium with the same local optical characteristics as a finite layer under study. Eq. (4.15) accounts for the influence of light absorption on the reflection function of clouds. Clearly, the reflection function decreases if additional absorbers are present in cloud droplets.

Constants l and m are defined by the following integrals(van de Hulst, 1980):

$$l = 2\int_{-1}^{1} i^2(\eta)\eta d\eta, \qquad (4.16)$$

$$m = 2\int_{0}^{1} K(\eta) i(-\eta)\eta d\eta, \qquad (4.17)$$

$i(\eta)$ being the angular distribution of light in deep layers of a cloud , where so-called asymptotic regime takes place (Sobolev, 1972).

Functions $R_\infty(\mu,\mu_0,\varphi)$, $K(\mu)$ and constants *m* and *l* have the following asymptotic forms when light absorption by droplets is relatively small($\omega_0 \to 1$) (van de Hulst, 1980):

$$R_\infty(\mu,\mu_0,\varphi) = R_\infty^0(\mu,\mu_0,\varphi) - 4\sqrt{\frac{1-\omega_0}{3(1-g)}}K_0(\mu)K_0(\mu_0), \quad (4.18)$$

$$K(\mu) = K_0(\mu)\left(1-2\alpha\sqrt{\frac{1-\omega_0}{3(1-g)}}\right), \quad (4.19)$$

$$m = 8\sqrt{\frac{1-\omega_0}{3(1-g)}}, \quad (4.20)$$

$$l = 1-4\alpha\sqrt{\frac{1-\omega_0}{3(1-g)}}, \quad (4.21)$$

and $\gamma \to \sqrt{3(1-\omega_0)(1-g)}$ as the single scattering albedo $\omega_0 \to 1$. Thus, at $\omega_0 = 1$: $R_\infty = R_\infty^0$, $K(\mu) = K_0(\mu)$, $m=l=0$ and Eq. (4.15) transforms into Eq. (4.6).

Note, that Eqs. (4.18) – (4.21) follow from the asymptotic analysis of the radiative transfer equation. The integration of Eq.(4.18) with respect to all angles yields: $r_\infty = 1-y$, where $y = 4\sqrt{\frac{1-\omega_0}{3(1-g)}}$ and r_∞ is the spherical albedo of an absorbing semi-infinite cloud (van de Hulst, 1980). Thus, the parameter *y* can be interpreted as a fraction of photons, absorbed in a weekly absorbing semi-infinite cloud. It depends both on the single scattering albedo and the asymmetry parameter. Clouds having larger values of *g*, therefore, absorb more light. Larger values of *g* imply that photon scattering increases at small angles. Thus the photon path length before its escape from the medium is also increased. As a consequence, this results in increased light absorption in the medium. Media having different values of ω_0 and *g*, but the same values of y, have the same values of r_∞. The parameter *y* (devided by four) is called the similarity parameter(van de Hulst, 1980). It is a useful parameter, describing the optical properties of clouds. Substituting *y* into Eqs. (4.18) – (4.21) yields:

$$R_\infty(\mu,\mu_0,\varphi) = R_\infty^0(\mu,\mu_0,\varphi) - yK_0(\mu)K_0(\mu_0), \quad (4.22)$$

$$K(\mu) = K_0(\mu)\left(1-\frac{\alpha y}{2}\right), \quad (4.23)$$

$$m = 2y, \quad (4.24)$$

and

$$l = 1-\alpha y. \quad (4.25)$$

Eqs. (4.22) – (4.25) were derived assuming that $\omega_0 \to 1$. Alternatively, the right-hand sides of Eqs. (4.22) – (4.25) give us the first terms of the expansion of correspondent functions with respect to *y*. The accuracy of equations decreases with ω_0. The higher terms of the expansions are not known or quite complex (Minin, 1988). However, it has been shown that the following equations account approximately for higher order terms (Rozenberg, 1962; Zege et al., 1991; Kokhanovsky et al., 1998):

$$R_\infty(\mu,\mu_0,\varphi) = R_\infty^0 \exp\left(-yu(\mu,\mu_0,\varphi)\right), \tag{4.26}$$

$$mK(\mu)K(\mu_0) = \left(1\text{-e}^{-2y}\right)K_0(\mu)K_0(\mu_0), \tag{4.27}$$

$$l = \exp(-\alpha y), \tag{4.28}$$

where a viewing function is defined by

$$u(\mu,\mu_0,\varphi) = \frac{K_0(\mu)K_0(\mu_0)}{R_\infty^0(\mu,\mu_0,\varphi)} \tag{4.29}$$

and does not depend on ω_0 and τ. The viewing function has a small dependence on the microstructure of clouds (e. g., the droplet size distribution). This follows from the low sensitivity of the functions $R_\infty^0(\mu,\mu_0,\varphi)$, $K_0(\mu)$ in Eq. (4.29) on the microstructure of clouds (see Figs. 13, 14) .

It should be stressed that the approximations, which lead to Eqs. (4.26) – (4.28) , avoid the solution of integral equations (van de Hulst, 1980; Yanovitskij, 1997) for the determination of functions $R_\infty(\mu,\mu_0,\varphi)$ and $K(\mu)$ in Eq. (4.15). Eqs. (4.26), (4.28) yield the transform to exact asymptotic results (4.22) , (4.25) as $y \to 0$.

Substituting Eqs. (4.27) - (4.28) into Eq. (4.15), we have:

$$R(\mu,\mu_0,\varphi,\tau) = R_\infty(\mu,\mu_0,\varphi) - te^{-x-y}K_0(\mu)K_0(\mu_0), \tag{4.30}$$

a new parameter $x = \gamma\tau$ and the global transmittance *t* is given by:

$$t = \frac{\sinh y}{\sinh(\alpha y + x)} \tag{4.31}$$

Eq. (4.31) yields Eq. (4.7) at $\omega_0 = 1$. Note, that the value of *exp(- x)* describes the attenuation of light field in deep layers of a cloud (van de Hulst, 1980).

Eq. (4.30) was first proposed by Rozenberg(1962, 1967). However, his derivation differs from that presented here. Also he assumed that $\alpha = 1$, which is not consistent with the exact asymptotic result, given by Eq. (4.7).

The range of applicability of Eq. (4.26) with respect to higher values of y can be readily extended using the following simple correction:

$$R_\infty(\mu,\mu_0,\varphi) = R_\infty^0(\mu,\mu_0,\varphi)\exp\left(-y(1-cy)u(\mu,\mu_0,\varphi)\right). \tag{4.32}$$

where *c* = 0.05. The value of *c* was obtained by the parametrization of calculations with exact radiative transfer code (Mishchenko et al., 1999). The accuracy of Eqs. (4.30) - (4.32) (see Eqs. (4.5), (4.10), as well) for the wavelength $\lambda = 1.55\mu m$, where water weakly absorbs radiation, has been investigated and the results are presented in Figs. 15 a, b. It was assumed that the effective radius of droplets is equal to 6 micrometers and the parameter $\mu = 6$ in Eq. (2.3). The Mie calculations for this case yield: $\omega_0 = 0.9935$, $g = 0.8214$. It follows from Figs. 15 a, b that Eq. (4.30) is an accurate representation for $\tau \geq 5$. The error is less than 2.5 % for this case, which is a relatively small error, compared to the uncertainty in cloud model used (e. g., vertically and horizontally homogeneous cloud field). The constant error at $\tau > 30$

is mostly due to the error of the approximation for the reflection function of a semi-infinite cloud, given by Eq. (4.32).

The comparison of data for wavelengths 0.65 and 1.55 micrometers, presented in Fig. 15a, show us that the limit of the semi-infinite cloud is achieved more rapidly for infrared absorbing wavelengths. This result can be used in the estimation of the droplets size even if the optical thickness of clouds itself is not retrieved. The larger absorption , the more quicker the limit of the semi-infinite medium is reached. Interestingly, both curves in Fig. 15a cross around the optical thickness 10. At optical thicknesses lower than 10, the reflection function at the absorbing channel is higher. This is because of the differences in the phase function between the visible and infrared spectral regions. More detailed studies of accuracy of Eq. (4.30) were performed by Kokhanovsky et al. (1998).

Eq. (4.30) is used to determine the optical thickness and effective radius from spectral reflection function measurements over extended cloud fields (Kokhanovsky and Zege, 1996). Eq. (4.6) is obtained from Eq. (4.30) for $\omega_0 = 1$.

Surface reflection is accounted for by Eq.(4.12). Substitution of Eq.(4.30) into Eq. (4.12) yields:

$$\hat{R}\left(\mu,\mu_0,\varphi,\tau\right) = R_\infty\left(\mu,\mu_0,\varphi,\tau\right) - \left[\exp(-x-y) - \frac{t\,A}{1-A\,r}\right] T\left(\mu,\mu_0\right), \qquad (4.33)$$

where

$$T\left(\mu,\mu_0\right) = tK_0\left(\mu\right)K_0\left(\mu_0\right) \qquad (4.34)$$

is the transmittance function of cloud, r is the total reflectance of the cloud, t is given by Eq. (4.31) and R_∞ is given by Eq.(4.32). It is assumed that $tK\left(\mu\right)K\left(\mu_0\right) \approx tK_0\left(\mu\right)K_0\left(\mu_0\right)$. This assumption is valid for single scattering albedos close to one.

4.3 The total reflectance

Let us find the approximate solution for the total reflectance r in Eq. (4.33). Clearly, the value of $r \neq 1-t$ due to light absorption in a cloudy medium.

The total reflectance or the spherical albedo r is defined by (Sobolev, 1972):

$$r = \frac{2}{\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \cos\vartheta \int_0^{\pi} d\vartheta_0 \cos\vartheta_0 R\left(\vartheta_0,\vartheta,\varphi,\tau\right) . \qquad (4.35)$$

For the case of idealized semi-infinite nonabsorbing clouds (Sobolev, 1972) and as a result of the conservation of energy law,

$$\frac{2}{\pi}\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\vartheta_0 \cos\vartheta_0 \int_0^{\pi/2} d\vartheta \cos\vartheta R_\infty^0\left(\vartheta_0,\vartheta,\varphi\right) = 1 \qquad (4.36)$$

and

$$\frac{1}{\pi}\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\vartheta \cos\vartheta R_\infty^0\left(\vartheta_0,\vartheta,\varphi\right) = 1, \qquad (4.37)$$

i. e. all photons injected into a cloudy medium are reflected back in outer space after an infinite travel time. Here $R_\infty^0\left(\vartheta_0,\vartheta,\varphi\right)$ is the reflection function of a semi-infinite nonabsorbing cloud. The reflection function $R_\infty^0\left(\vartheta_0,\vartheta,\varphi\right)$ of a cloudy medium only weakly depends on its microstructure (see Fig. 13) and by by definition, it does not depend on either the optical thickness $\tau = \sigma_{ext} L$ or the single scattering albedo $\omega_0 = \sigma_{sca} / \sigma_{ext}$. Here σ_{ext} is the extinction coefficient and σ_{sca} is the scattering coefficient of a cloudy layer of the geometrical thickness *L*.

It follows from Eqs. (4.30) and (4.35) for absorbing clouds:

$$r = r_\infty - t\exp(-x-y), \quad (4.38)$$

where

$$r_\infty = \frac{2}{\pi}\int_0^{2\pi} d\varphi \int_0^{\pi/2} d\vartheta \cos\vartheta \int_0^{\pi/2} d\vartheta_0 \cos\vartheta_0 R_\infty\left(\vartheta_0, \vartheta, \varphi\right) \quad (4.39)$$

and we taking for the normalization condition(van de Hulst, 1980)into account:

$$2\int_0^1 d\mu \mu K_0\left(\mu\right) = 1. \quad (4.40)$$

The approximate formula (4.10) for the function $K_0\left(\mu\right)$ obeys the integral relation (4.40). The constant r_∞ represents the total reflectance of a semi-infinite layer. According to the definition (4.36), $r_\infty = 1$ at $\omega_0 = 1$. Eq. (4.39) is not readily analytically integrated at arbitraty values of ω_0. However, it follows from Eqs. (4.39) and (4.18) as $\omega_0 \to 1$ (see also the discussion in the previous Section):

$$r_\infty = 1 - y \ . \quad (4.41)$$

At larger values of *y*, using the same substitution as was used in the derivation of Eq. (4.28) from Eq. (4.25), we obtain approximately for the integral (4.39):

$$r_\infty = \exp(-y) \quad (4.42)$$

or (see Eq. (4.28)): $r_\infty = l^{1/\alpha}$.

Combining Eqs. (4.31), (4.38), (4.42), we have for the total reflectance of a cloud layer:

$$r = \frac{\sinh(x+\alpha y)\exp(-y) - \sinh(y)\exp(-x-y)}{\sinh(x+\alpha y)} \ . \quad (4.43)$$

The substitution of Eq. (4.43) into Eq. (4.33) enables the reflection function of a cloud and underlying surface to be calculated.

The total light absorptance in a cloud layer is given by: *a =1- r- t*, where *r* is calculated by (4.43) and *t* by (4.31). As a result, we have the following analytical equation for the total light absorbtion inside a plane-parallel cloud having a finite thickness:

$$a = \frac{\sinh(x+\alpha y)(1-\exp(-y)) - \sinh(y)(1-\exp(-x-y))}{\sinh(x+\alpha y)} \quad (4.44)$$

It follows as $\alpha \equiv 1$:

$$r = \frac{\sinh(x)}{\sinh(x+y)}, t = \frac{\sinh(y)}{\sinh(x+y)}, a = \frac{\sinh(x+y) - \sinh(x) - \sinh(y)}{\sinh(x+y)}, \quad (4.45)$$

which yields the well-known formulae, presented elsewhere(see, e.g., Zege et al. (1991)).

Overall, the global radiative characteristics of cloudy media are well described by only two parameters: $x = \tau \sqrt{3(1-\omega_0)(1-g)}$ and $y = 4\sqrt{\frac{1-\omega_0}{3(1-g)}}$. The parameter x describes the attenuation of a light field in deep layers of semi-infinite weakly absorbing media. For the light intensity in deep layers: $I(\mu,\tau) = \psi(\mu)\exp(-x)$, where the angular distribution of light field $\psi(\mu)$ does not depend on the optical depth τ. We see from Eq.(4.42) that $y = \ln\left(\frac{1}{r_\infty}\right)$. Thus, the radiative characteristics of optically thick cloud layers are determined by parameters x and y, which govern light reflection and asymptotic regime for semi-infinite turbid media.

5. Satellite remote sensing of cloudy media

5.1 The optical thickness

Equations, presented in the previous section can be used for a rapid estimations of the radiative characteristics of cloudy media. They can be used also to check the accuracy of new algorithms, based on the numerical solution of the radiative transfer equation. Clearly, the numerical solutions and results, presented above , should converge as $\tau \to \infty$ and $\beta \to 0$.

Kokhanovsky and Macke(1999) used these approximations to study the influence of the shape of particles on the radiative transfer in clouds. They found, e.g., that clouds with nonspherical particles are more reflective (larger values of the reflectance) as compared to clouds with spherical droplets with the same value of the volume to the particle surface area ratio. The opposite is true for the transmittance.

However, the most important area of the application of these solutions lays in the area of remote sensing and inverse problem solutions(Zege and Kokhanovsky, 1996). In particular, this approach allows to avoid or reduce (if thin clouds are also under consideration) the pre-calculation and storage of so-called look-up tables(Arking and Childs, 1985; Rossow et al., 1989; Nakajima and King, 1990).

In particular, we have for the global transmittance from Eq. (4.14) after simple algebraic calculations:

$$t = \frac{(1-A)\Lambda}{1-A\,(1+\Lambda)}, \tag{5.1}$$

where the function Λ is introduced and given by

$$\Lambda \equiv \Lambda(\mu,\mu_0,\varphi) = \frac{R_\infty^0(\mu,\mu_0,\varphi) - \hat{R}\,(\mu,\mu_0,\varphi,\tau)}{K_0(\mu)K_0(\mu_0)}. \tag{5.2}$$

The analytical results for functions $R_\infty^0(\mu,\mu_0,\varphi)$ and $K_0(\mu)$ have been presented above. Thus, the global transmittance t , and, correspondingly, the total reflectance $r=1-t$ can be obtained from Eqs. (5.1) and (5.2), and a knowledge of the surface albedo A and the measured value $\hat{R}\,(\mu,\mu_0,\varphi,\tau)$.

For such a retrieval one does not need to know the optical thickness of clouds and the average size of droplets. This is an extremely important point for climate studies, where the global and temporally averaged value of the cloud reflectance *r=1-t* is an important parameter. It follows that $r < 0.8$ for natural water clouds in visible (Danielson, 1969), which implies that clouds with optical thicknesses larger than 70-100 do not appear. This is not the case in reality (Trishchenko et al., 2001). The paradox is not resolved so far. Most probably the reduced reflectance is related to aerosol absorption in clouds. The inhomogenity and finite size of clouds also may play a role in this effect.

Let us consider Eq. (5.1) in more detail. First of all, $t \equiv \Lambda$ at *A=0,* secondly, at *A=1*: *t=0* and *r=1*. This shows that all photons incident on optically thick clouds over bright surfaces survive and return back to outer space. They yield no information about actual cloud thickness. This explains why the retrieval of cloud parameters over bright surfaces (e.g., snow and ice) can be hardly performed in visible(Platnick et al., 2001). The information on the global transmittance *t* can be used to find the scaled optical thickness(Rozenberg, 1978; King, 1987), given by

$$\tau^* = \tau(1-g) \ . \tag{5.3}$$

It follows from Eqs. (4.7) and (4.49) that

$$\tau^* = \frac{4}{3}\left[t^{-1} - \alpha\right], \tag{5.4}$$

where *t* is given by Eq. (5.1). Again the value of τ^* can be obtained although there is no information about the size of droplets and the actual optical thickness of clouds.

Eq. (5.4) can be used for the retrieval of τ^* from the measurement $\hat{R}(\mu,\mu_0,\varphi,\tau)$ at a single wavelength. The functions $R_\infty^0(\mu,\mu_0,\varphi)$ and $K_0(\mu_0)$ in Eq. (5.2) and the parameter α in Eq. (5.4) are defined by Eqs. (4.5), (4.10), and (4.11) respectively .

Eq. (5.1) can be used for the derivation of the optical thickness τ (see Eq. (5.3)) if the value of *g* is known(Rossow, 1989). It is around 0.74 for ice clouds as it was discussed earlier. However, for warm clouds the asymmetry parameter g depends on the size of droplets even for nonabsorbing channels (see Eq. (3.35)). Often the dependence $g(a_{ef})$ is neglected and it is assumed that $a_{ef} = 10\mu m$ for water clouds (Rossow and Schiffer, 1999). Then it follows from Eq. (3.36): *g = 0.86* at $\lambda = 0.65\mu m$ and $a_{ef} = 10\mu m$. Utilising Eq. (5.3), (5.4), this value of *g* can be used for a crude estimation of the optical thickness of liquid clouds. Clearly, errors can be introduced, if one assumes the fixed *a priori* defined value of *g* . It follows from Eq. (3.36) at $\lambda = 0.65\mu m$ that *g* = 0.84 - 0.87 at a_{ef} = 4 - 20 μm . From Eq. (5.4) we have: $\tau = H\tau^*$, $H \equiv (1-g)^{-1} \approx$ 6.3 – 7.6 and $\tau \in [9.4, 11.5]$ at $\tau^* = 1.5$, depending on the value of *g* used. The assumption that $a_{ef} = 10\mu m$ yields : *g = 0.86* and *H=7.2*, $\tau = 10.7$. This leads to the relative error *7-14 %* in the retrieved optical thickness (i. e., a range of possible values from $\tau = 9.4$ to $\tau = 11.5$ instead of $\tau = 10.7$). This uncertainty in the optical thickness determination can be removed if measurements in the near infrared region of the electromagnetic spectrum are performed, enabling the size of droplets and, therefore, the asymmetry parameter *g* to be estimated. For this, however, we should be sure that we have a liquid and not ice or a mixed phase cloud.

5.2 The size of droplets

As it was specified above for the correct estimation of the optical thickness of clouds from space we need to know the effective radius of droplets. The size of droplets can be found if the reflection function in near-infrared region spectrum is measured simultaneously(Nakajima and King, 1990; Kokhanovsky and Zege, 1996). This is due to the fact that the reflection function in the infrared strongly depends on the probability of photon absorption by droplets. This probability is proportional to the effective radius of droplets, as it was discussed before(see Eq. (3.48)).

The influence of absorption and scattering of light by molecules and aerosol particles on the measured value $R(\mu,\mu_0,\varphi,\tau)$ is often neglected in the cloud retrieval algorithms. However, correction can be easily taken into account if needed (Wang and King, 1997; Goloub, 2000). The influence of the surface reflection on the cloud reflection function, assuming that the surface is Lambertian with albedo *A*, is easily taken into account. Then it results (see Eqs. (4.14), (4.33)):

$$\hat{R}_1\left(a_{ef},w\right)=R_\infty^0-\frac{t_1\left(a_{ef},w\right)\left[1-A_1\right]}{1-A_1\left[1-t_1\left(a_{ef},w\right)\right]}K_0\left(\mu\right)K_0\left(\mu_0\right), \tag{5.5}$$

$$\begin{aligned}\hat{R}_2\left(a_{ef},w\right)=R_\infty^0\exp\left(-y\left(a_{ef}\right)(1-cy\left(a_{ef}\right))u\right)\\-\left[\exp(-x\left(a_{ef},w\right)-y\left(a_{ef}\right))-\frac{t_2\left(a_{ef},w\right)A_2}{1-A_2r_2\left(a_{ef},w\right)}\right]t_2\left(a_{ef},w\right)K_0\left(\mu\right)K_0\left(\mu_0\right)\end{aligned} \quad . \tag{5.6}$$

The subscripts " 1 " and " 2 " refer to wavelengths λ_1 and λ_2 in visible and near – infrared channels respectively. The values of A_1 and A_2 give us the surface albedos in visible and near-infrared. The explicit dependence of functions involved on the parameters a_{ef} and w to be retrieved is introduced in brackets. The liquid water path w is preferred to the optical thickness in retrieval procedures due to the independence of w on the wavelength. The optical thickness is uniquely defined if a_{ef} and w are known.

The equations (5.5) and (5.6) form a nonlinear system of two algebraic equations having two unknowns (a_{ef} and w). Standard methods and programs are available to solve this system. In particular, we can find the value of w from Eq. (5.5) analytically. The substitution of this result in Eq.(5.6) gives us a single transcendent equation for the effective radius of droplets determination(Zege and Kokhanovsky, 1996).

5.3 The thermodynamic state of clouds

The discrimination of liquid water and ice clouds is of importance for many applications, including flight safety and Earth climate studies. The size and shape of particles in warm and ice clouds are different. This influences the energy transmitted and reflected by a cloud.

The discrimination can be performed, taking into account the difference in angular or spectral distribution of reflected light. We present results of calculation of the reflection function of cloudy media with liquid and frozen water droplets in Fig. 17. It follows from this Figure that minima in the reflection function of ice clouds (e.g., near 1.5 and 2.0 μm) are moved to larger wavelengths as compared to the case of liquid droplets. This is, of course, due to the different in the spectral behaviour of imaginary parts of the complex refractive index of liquid water and ice. Note, that minima for liquid water also moved to larger wavelengths as compared to the absorption bands of water vapour. These different positions of minima can be easily registered with modern spectrometers (see, e.g., Bovensmann et al., 1999).

Another possibility is to consider different angular behaviour of the reflection function for ice and water clouds at specific scattering geometries(e.g., rainbow , glory and halo scattering). In particular, the reflection function of water clouds has a maximum near the rainbow scattering angle. This is not the case for ice clouds, which can be also easily detected. This feature becomes even more pronounced if the degree of polarization at the rainbow geometry is studied(Goloub et . al., 2000; Kokhanovsky, 2000).

5.4 The cloud height and cloud fraction

Another important characteristic of a cloud is its height. It can be retrieved, using active remote sensing techniques, which are based on the analysis of data from space-borne lidars(Winker and Trepter, 1998). Passive measurements also can be used. For instance, Yamamoto and Wark(1961) proposed to use the oxygen A band, centered at 0.761 μm . The physical basis of this method is simple. Indeed, the reflection function has a deep minimum around 0.761 μm due to oxygen absorption there. This minimum is not shown in Fig. 17, because only scattering and absorption of light by cloud particles was accounted for in the calculation for this figure. Clearly, the depth of the absorption line will depend on the cloud height. Indeed, photons can hardly penetrate thick clouds and be absorbed by the oxygen in the air column below the cloud. This will lead to the increase in the value of the reflection function at 0.761 μm for the case of clouds at high altitudes. We see, thus, that the depth of the absorption line is larger for low clouds. The practical application of the method, however, is not so simple(Kunze and Chance, 1994; Koelemeijer et al., 2001). First of all, the depth of line also depends on the the oxygen absorption cross section. The cross section varies, depending on temperature and pressure. Thus, one should use *a priori* assumptions on the temperature and pressure variation with height in the Earth atmosphere. The generally unknown surface albedo can also influence the retrieval accuracy. Other possible sources of errors are described in detail by Kuze and Chance (1994) and Koelemeijer et al. (2001). The largest complication arises for pixels, which are only partially covered by clouds. One possibility is to ignore them altogether. However, this will lead to a big reduction of data. To overcome this problem Koelemeijer et al. (2001) proposed the algorithm, which sameltaneosly retrieve cloud top height/pressure and cloud fraction. Note, that global information on cloud fraction/cover F is of considerable importance by itself (Batey et al. 2000). For instance, Minnins et al.(2001) found, analysing data of the experiment, performed over Arctic Ocean, that the value of F varies in the range 0.55 0.85, depending on the exact region under study. Mean cloud amounts were near 70%(Minnins et al., 2001). Globally, cloud cover fractions are somewhat below this number. However, in any case they larger than 0.5(Liou, 1992). This once more underlines the importance of clouds in the radiation balance and atmosphere heating rates studies.

It is interesting to note that the global cloud cover increased during the past century (Palle Bago and Buttler, 2001). This argues against a dominating role by solar activity (via galactic cosmic rays) over cloud formation.

5.5 The remote sensing of crystalline clouds

The remote sensing of optical thicknesses and effective size of droplets in crystalline clouds is complicated by their low optical thickness (usually smaller than 5), the high spatial and temporal variability of cloud properties and nonspherical shape of particles. If the thickness of a crystalline cloud is high, then the optical thickness can be found in the same way as it was discussed above, assuming the asymmetry parameter, equal to 0.74 (see Eqs. (5.1)-(5.4)). The problem is to find the correspondent reflection function of a semi-infinite medium(see Eq. (5.4)). One possibility is to calculate it beforehand, using the phase function of a fractal ‘’fictive’’ scatterer.

For thinner clouds one should bild the pre-calculated table of reflection functions, which should be compared with experimental data to establish both optical thickness of clouds and the size/shape of droplets(Masuda et al., 2002). This is not an easy problem. In particular, the model of ice spheres can not be used for this purpose (Mishchenko et al.(1995, 1996); Chepfer et al., 1998; Rolland et al.(2000); Doutriaux-Boucher et al.(2000)).

Yang et al. (2001) considered the influence of ice crystals habits and crystal dimensions vertical variability on the satellite cloud retrieval algorithms. It was accounted for more complex shapes and larger sizes of crystals near the base of the cloud as compared to its top, where crystals are smaller and more rounded. The authors state that the vertical distribution of optical characterstics of clouds can be neglected if visible channels are used, but the vertical inhomogenity should be fully accounted for if one is interested in the average size of particles retrievals (e.g., from the measurements of the cloud reflection function at 2.11 μm Moderate Resolution Imaging Spectrometer channel (King et al., 1992)). The information on the vertical structure of a crystalline cloud is not known *a priori*. This complicates the retrieval procedure. We see, therefore, that the creation of a suitable look-up table for crystalline clouds, is not at all a trivial problem.

6. Inhomogeneous clouds

6.1 Vertical inhomogenity

Vertical and horizontal inhomogenity of clouds can be dealt with in the framework of the Monte-Carlo methods of the radiative transfer equation solutions(Scheirer and Macke, 2001). This is of considerable importance for crystalline media, as it was discussed in the previous section. Unfortunately, Monte-Carlo methods are extremely slow and can not be applied in the operational satellite cloud retrieval algorithms.

The account for the vertical inhomogenity, however, can be easily done in the framework of the theory of optically thick layers, discussed above. Correspondent equations are presented by Sobolev (1972) and Yanovitskij(1997). For the sake of simplicity, we consider here only the case of clouds in visible, where light absorption can be neglected. Then the reflection function of a vertically inhomogeneous optically thick cloud can be presented by Eq. (4.6) as in the case of a homogeneous layer. The meaning of parameters in this equation becomes different, however. Namely, τ is the optical thickness of a vertically inhomogeneous cloud, R_∞^0 is the reflection function of a semi-infinite cloud with the same vertical distribution of optical characteristics as a finite cloud and g is the average asymmetry parameter. It should be stressed that the value of g does not change with the size of droplets considerably. So it can be assumed to be equal to some *a priori* defined value(say, e.g., 0.86

(Rossow and Schiffer, 1999)). Then, accounting also for the weak sensitivity of the reflection function R_{∞}^{0} to the size of droplets , we state that Eq.(4.6) can be also applied to the derivation of the optical thickness of vertically inhomogeneous clouds. This suggest that one can not retrieve the vertical distribution of extinction coefficient in a cloud from the reflection function in visible.

The situation in the near-infrared is not so simple. Here the derived size of droplets depends on the average photon penetration depth, which is, of course, the function of the wavelength(Platnick, 2000). It means, that the derived effective size of droplets is the function of the wavelength. Generally, the absorption increases with the wavelength. So the penetration depth decreases with the wavelength. Thus, we arrive to the conclusion that reflection functions larger wavelength (e.g., 2.11 μm) will give us values of the droplets radii closer to the cloud tops and reflection functions at smaller wavelengths (e.g., 1.6 μm) give us the radius of droplets deep inside clouds. These different values of radii are considered as a shortcoming of the retrieval method for vertically inhomogenious clouds. On the other hand, this opens a new possibility to study the vertical distribution of droplets sizes inside the cloud, analysing spectral reflectances of clouds.

In conclusion, we note, that radii of droplets increase with height. It means that values of radii, retrieved at 1.6 μm should be smaller than those obtained from the channel at 2.11 μm . This is also observed experimentally(Platnick et al., 2001).

6.2 Horizontal inhomogenity.

The real cloud fields are horizontally inhomogeneous. They also have complex shapes, which complicates the theoretical modeling (Rogovtsov, 1991; 1999). This also produces a unphysical angular dependence of the cloud optical thickness for different solar angles (Loeb and Davies, 1996, Loeb and Coakley, 1998) if the model of the horizontally inhomogeneous layer is used in the retrieval procedure for inhomogeneous cloud fields. Fouilloux et al. (2000) found that derived cloud optical thicknesses and effective radii for inhomogenious clouds depend on the averaging scale. Therefore, they state, that comparisons between aircraft measurements and satellite observations not be valid for heterogeneous clouds(which is the case for most of the clouds). This puts the validation of cloud satellite products with airborne radiometers in question.

Approximations for horizontal photon transport within real-world clouds were developed by Platnick(2001). In particular, he derived analytic approximations for the root-mean-square horizontal displacement of reflected and transmitted photons, relative to the incident cloud-top location. Usually the influence of the horizontal inhomogenity of clouds on their radiative charactersitics is studied in the framework of the independent column (or pixel) approximation (Cahalan et al., 1994). This approximation neglects the horizontal photon transport between adjacent columns. Then one can obtain for the reflection function of a pixel in the framework of this approximation(Pincus and Klein, 2000):

$$\overline{R} = \int_{0}^{\infty} R(\tau) f(\tau) d\tau , \qquad (6.1)$$

where $f(\tau)$ is the optical thickness distribution function for a given pixel and $R(\tau)$ is the reflection function for a horizontally homogeneous cloud with a given optical thickness τ . The accuracy of this approximation was studied by Cahalan et al. (1994), Fu(2000) and

Scheirer and Macke(2001). It was found that this approximation has a high accuracy for the domain-averaged radiative fluxes.

It is known (Cahalan et al., 1994) that $\bar{R} < R(\bar{\tau})$, where $\bar{\tau}$ is the average optical thickness. It means that the values of $\bar{\tau}$, obtained, from measurements over horizontally inhomogenious clouds in the assumption of a horizontally homogeneous plane-parallel clouds, are underestimated. So the correction of the optical thickness obtained by an empirical factor is needed (Rossow and Schiffer, 1999).

It is difficult to apply Eq.(6.1) to the radiative transfer problem analysis due to the necessity to perform calculations $R(\tau)$ many times for a given pixel. The problem can be greatly simplified if one uses approximate analytical solutions, which are valid for optically thick clouds, together with analytical forms for the function $f(\tau)$, which is usually given by the gamma (Barker, 1996) or the lognormal (Nakajima et al., 1991) distributions. The lower limit of integration should be set equal to 5 in this case. The integral in the range from 0 to 5 can be estimated using the exact radiative transfer equation. This contribution is, however, small at large values of $\bar{\tau}$, where is can be neglected, providing the possibilty of analytic integration in Eq. (6.1).

7. Conclusion

The main idea of this review was to consider various approaches to calculate local and global optical charactersitics of cloudy media. Main tools reviewed are geometrical optics approximation and the theory of optically thick turbid layers. Both theories can be substituted by exact solutions (the Mie theory for local optical chatacteristics and the radiative transfer theory for the radiative charactersitics) in the case of homogeneous clouds with spherical particles. However, they appear to be very important in bringing the forward propagation model closer to the reality(e.g., nonspherical shape of crystals, effects of cloud inhomogenity). They allow to consider cases, which difficult or impossible to handle with exact techniques. Another important feature of the approximate methods is the possibillity to simplify inverse problems of cloud optics.

The polarization of light by cloud droplets and crystals was not considered in detail here. However, it should be stressed that the account for light polarization brings us new possibilities to detect and characterize cloudy media (Hansen and Hovenier, 1974; Wauben, 1992; Goloub,2000; Kokhanovsky, 2000, 2001a,b).

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Figure captions

Fig.1 Real part of the refractive index of water and ice.
Fig.2 Imaginary part of the refractive index of water and ice.
Fig.3 The extinction coefficient of a cloudy medium with water droplets, characterized by the PSD (2.3) at $a_{ef} = 4\mu m$, $\mu = 6$. The value of C_w is equal to 0.1g/m^3.

Fig.4 The spectral dependence $B(\lambda)$, obtained for the same conditions as in Fig.3.

Fig.5. The probability of photon absorption, obtained for the same conditions as in Fig. 3. The results for the effective radius $a_{ef} = 16\mu m$ are also shown.

Fig.6. The error of the geometrical optics approximation for the probability of photon absorption, obtained from data, presented in Fig.5 for effective radii 4 and 16 micrometers. Data for the effective radius 6 micrometers are also shown.
Fig. 7. Phase functions of water clouds, obtained from the Mie theory for the same conditions as in Fig.3. The data for a_{ef} equal to 6 μm and 16 μm are also given for the comparison.

Fig. 8. The coefficients a_s, obtained for the same conditions as in Fig.3. Data for a_{ef} equal to 6 μm are also shown.

Fig.9. The spectral dependence $C(\lambda)$, obtained for the same conditions as in Fig.3.

Fig.10. The asymmetry parameter, obtained for the same conditions as in Fig. 3. The results for the effective radius $a_{ef} = 6\mu m$ are also shown.

Fig.11. The error of the geometrical optics approximation for the asymmetry parameter, obtained from data, presented in Fig.10 for effective radii 4 and 6 micrometers.
Fig.12 . Phase functions of hexagonal ice cylinders with the aspect ratio(length/size of the side of the hexagonal cross section) equal to 5.88 and fractal particles in random orientation at the wavelength 0.5 μm. Only the geometrical optics contribution of both phase functions is shown.
Fig. 13. The reflection function of an idealized semi-infinite nonabsorbing cloud $R_\infty^0(0, \vartheta_0, 0)$ obtained from the exact radiative transfer code (Mishchenko et. al., 1999) and approximation (4.5) at the wavelength $\lambda = 0.65\mu m$ and the effective radii of droplets $a_{ef} = 6$ and $16\mu m$. It is assumed that particles in a cloud are characterized by the gamma particle size distribution(2.3) with the parameter $\mu = 6$.
Fig.14. The escape function, calculated with exact radiative transfer code for the Heyney-Greenstein phase function at g = 0.75, 0.8 and 0.9 (Yanovitskij, 1997) and with approximation, given by Eq. (4.10).
Fig. 15a. The dependence of the reflection function of a cloudy layer on the optical thickness according to Eqs. (4.6) (at λ=0.65μm) and (4.30) (at λ=1.55μm) for $a_{ef} = 6\mu m$, $\vartheta = 7^\circ$, $\vartheta_0 = 49^\circ$, φ=0° as compared to exact radiative transfer computations. It is assumed that particles in a cloud are characterized by the gamma particle size distribution (2.3) with the parameter $\mu = 6$.
Fig 15b. The errors of approximations , given by Eqs. (4.6) (at λ=0.65μm) and (4.30) (at λ=1.55μm) for $a_{ef} = 6\mu m$, $\vartheta = 7^\circ$, $\vartheta_0 = 49^\circ$, φ=0° as compared to exact radiative transfer computations.

Fig.16. The frequency of registration of different optical thicknesses of cloudy media, obtained from satellite and ground measurements as presented by Trishchenko et al.(2001)

Fig.17. The spectral dependence of the reflection function of cloudy media for the nadir observation and the solar angle equal to 60 degrees. Clouds are composed of water or ice spherical particles with the effective radius 6 μm. It is assumed that particles in a cloud are characterized by the gamma particle size distribution (2.3) with the parameter $\mu = 6$. The geometrical thickness of cloud is equal to 500m. The liquid water path equal to $100g/m^2$, which gives the optical thickness equal to 27 at the wavelength 0.55 μm. Computations of local optical characteristics were performed, using Eqs. (3.11), (3.23), (3.37). The reflection of light from surface, scattering and absorption of light by aerosols and gases are neglected.

Table 1. Droplet size distributions

f(a)	B	$< a >$	a_{ef}	Δ	Δ_{ef}	$\langle a^n \rangle$
Gamma distribution $Ba^{\mu} e^{-\mu \frac{a}{a_0}}$	$\frac{\mu^{\mu+1}}{a_0^{\mu+1}\Gamma(\mu+1)}$	$a_0\left(1+\frac{1}{\mu}\right)$	$a_0\left(1+\frac{3}{\mu}\right)$	$\sqrt{\frac{1}{\mu+1}}$	$\frac{1}{\mu+3}$	$\left(\frac{a_0}{\mu}\right)^n \frac{\Gamma(1+\mu+n)}{\Gamma(1+\mu)}$
Lognormal distribution $\frac{B}{a}\exp\left(-\frac{\ln^2 \frac{a}{a_m}}{2\sigma^2}\right)$	$\frac{1}{\sqrt{2\pi}\sigma}$	$a_m e^{0.5\sigma^2}$	$a_m e^{2.5\sigma^2}$	$\sqrt{e^{\sigma^2}-1}$	$e^{\sigma^2}-1$	$a_m^n e^{\frac{n^2\sigma^2}{2}}$

Table 2. Typical range of values N, C_v, and C_w in water clouds

N, cm^{-3}	C_v	C_w, g/m^3
20-1000	$10^{-7}-10^{-6}$	0.01 - 1

Table 3. Parameters b_i, β_i, and θ_i

i	b_i	β_i	θ_i
1	1744.0	1200.0	0.0
2	0.17	75.0	2.5
3	0.30	4826.0	π
4	0.20	50.0	π
5	0.15	1.0	π