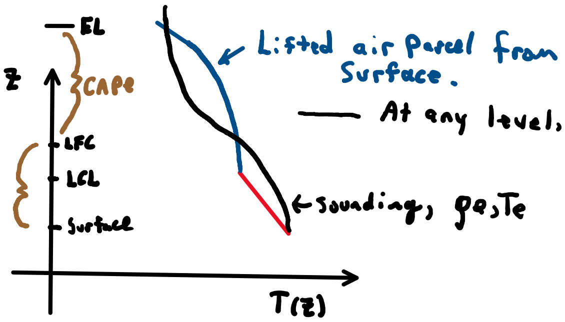
Cape Equation 1

Sunday, October 15, 2023 4:08 PM

Convective available Potential energy

Convective inhibition



ρ_p, T_p

Virtual temperature

—— At any level, $P = \rho_e R_d T_e = \rho_p R_d T_p$

$$\frac{\rho_e}{\rho_p} = \frac{T_p}{T_e}$$

Buoyant force

Newton's 2ND Law:

$$\rho_p \frac{dw}{dt} = \rho_p\left(\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z}\right) = \rho_p w \frac{\partial w}{\partial z} = (\rho_e - \rho_p) g$$

Convective derivative

So

$$w\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}\left(\frac{w^2}{2}\right) = \left(\frac{\rho_e - \rho_p}{\rho_p}\right) g = \left(\frac{\rho_e}{\rho_p} - 1\right) g = \left(\frac{T_p - T_e}{T_e}\right) g$$

$$\partial\left(\frac{w^2}{2}\right) = \text{Change in K.E.} = \left(\frac{T_p - T_e}{T_e}\right) g\,\partial z = \text{Work done by the buoyant force.}$$

Integrating:

$$\int_{LFC}^{El} d\left(\frac{w^2}{2}\right) \doteq \frac{W_{max}^2}{2} = \int_{LFC}^{El} \frac{T_p - T_e}{T_e} g\,dz = CAPE$$

$$\boxed{W_{max} = \sqrt{2\,CAPE}}$$