Problem 1.20

Monday, September 12, 2022 7:36 AM

1.20 The equatorward flow in the trade winds averaged around the circumference of the Earth at 15 °N and 15 °S is ~1 m s^{-1}. Assume that this flow extends through a layer extending from sea level up to the 850-hPa pressure surface. Estimate the equatorward mass flux into the equatorial zone. [**Hint**: The equatorward mass flux across the 15 °N, in units of kg s^{-1}, is given by

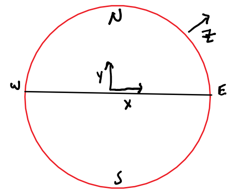
$$-\oint_{15\,°\mathrm{N}} \int_0^{z_{850}} \rho v\,dz\,dx$$

where ρ is the density of the air, v is the meridional (northward) velocity component, the line integral denotes an integration around the 15 °N latitude circle, and the vertical integral is from sea level up to the height of the 850-hPa surface.] Evaluate the integral, making use of the relations

$$\oint_{15\,°\mathrm{N}} dx = 2\pi R_E \cos 15°$$

and

$$\int_0^{z_{850}} \rho\,dz = \frac{(1000 - 850)\ \mathrm{hPa} \times 100\ \mathrm{Pa/hPa}}{g}.$$



(X, Y, Z) Spatial coordinates

(U, V, W) wind coordinates

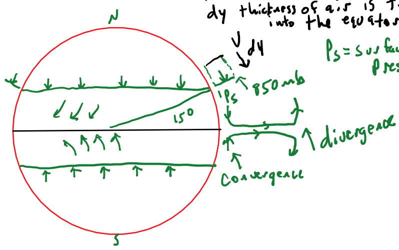
W — Vertical velocity, + values, updrafts

V — Meridional wind, V > 0 Southerly wind from the south

U — Zonal wind, U > 0, westerly, from the west

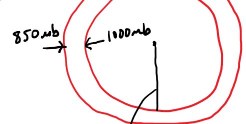
In time dt, dy thickness of air is transported into the equatorial region

How much mass per unit time is carried past Equator ± 15° Lat by trade winds?



Ps = surface pressure

Ring of flow at 15°N latitude



R = Re cos 15°

Variables:

V = flow in the y direction

|V| = 1 m/s

M = total mass into the ring

ρ(z) = air density.

$$\text{Mass} = m = \oint dx \int_{z=0m}^{850mb} \underset{kg/m^3}{\rho(z)}\,dz\,\underset{\sim}{dy} \quad V dt$$

1000 mb

1 ring North and South → 2 · 2πRe cos15°

Hydrostatic Approximation

$dP = -\rho g\,dz \qquad \frac{dP}{g} = -\rho\,dz$

$$\frac{1}{g}\int_{850mb}^{1000mb} dP$$

$$\boxed{\dot{m} = \text{mass flux} = \frac{\text{mass}}{\text{time}} = \frac{m}{dt} = \frac{4\pi R_e \cos 15°\, V\, \Delta P}{g}}$$

ΔP: 100000PA − 85000

Units $\dfrac{\cancel{m}\ \frac{\cancel{m}}{s}\ \frac{\cancel{N}}{\cancel{m^2}}}{\cancel{N}/kg}$ $\dfrac{kg}{s}$

$$\boxed{\frac{M}{dt} = 1.18 * 10^{11}\ \frac{kg}{\text{second}}}$$

Let τ = time for the entire atmosphere Mass to circulate through the trade winds

$$\tau = \frac{\text{Mass of Atmosphere} \sim 5 \times 10^{24}\ kg}{\dot{m}}$$

$$\boxed{\tau = 509\ \text{days} = 1.4\ \text{years}}$$