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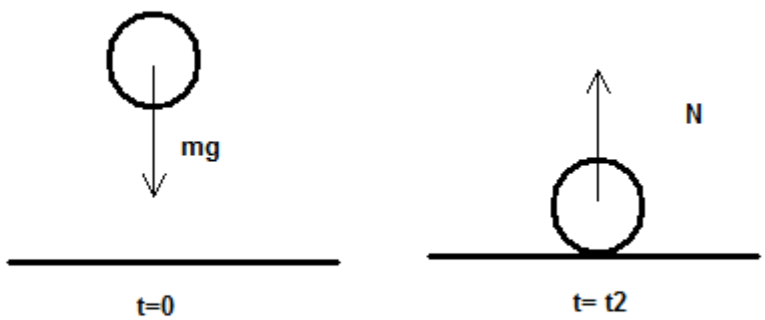
ATMS 611
Hmwk #2

1.15: ➔ *Prove that in the time average over an integral number of bounces, the downward force exerted by the ball upon the surface is equal to the weight of the ball.*



- the initial force on the ball at release (t=0) is *mg*. Its momentum is *mv*.
- at the point of impact v is at its maximum, and therefore momentum is maximum
- the force applied to the surface during each bounce is equal to the change in momentum over the time between bounces:

- $F_{bounce} = \frac{\Delta(mv)}{\Delta t} = \frac{2mv_b}{\Delta t}$, where v_b is the velocity at the point of impact, and Δt is the time between successive bounces (or also the time the ball takes to bounce and return to its initial elevation) (perfectly elastic assumption imposes the condition of perfect reflection of velocity vector, so that the initial return velocity equals the impact velocity)

- $v_b = \int_0^{t_b} g dt = gt_b$, the bounce velocity is dependent on t_b, which is the time from the initial elevation of the ball to the point of impact.

- $F_{bounce} = \frac{2mv_b}{\Delta t} = \frac{2mgt_b}{2t_b} = mg$

- $F_{surf} = \frac{1}{t}\sum_{n=1}^{t}(mg)_i = mg$ thus, the time averaged force over successive bounces is the weight force of the ball on the surface.

➔ This result leads to the conclusion that the weight of the atmosphere "felt" at the surface is due to pressure exerted on the surface from the kinetic energy of the gas molecules in the column. Although each individual gas molecule isn't bouncing up and down on the surface, the molecules adjacent to the surface are and the molecules just above them are bouncing on the atmospheric molecules adjacent to the surface. Thus, the kinetic energy of the column is effectively transferred downward and realized as mean force per unit area on a surface at its base (which is the definition of pressure). The random trajectories of the molecules are not exactly up-and-down either, but on average half of their momentum vector will be parallel to the vertical alignment of the column.

1.19: calculate scale height using the exponential density equation:

a) $\rho = \rho_o e^{\frac{-z}{H}}$, assume that the atmospheric density decreases exponentially with height

b) integrate the equation from the surface to infinity to find the atmospheric mass per unit area:

$$m_a(kg/m^2) = \int_0^\infty \rho_o e^{-z/H} = \rho_o \left[-He^{-z/H} \right]_{0|}^{\infty} = \rho_o [0 + H] = \rho_o H = \left[\frac{kg}{m^3} m \right]$$

c) the pressure at the surface is the force per unit area due to the weight of the atmospheric mass:

$$P\left(\frac{kg}{ms^2} \right) = \left[\frac{kg}{m^2} \frac{m}{s^2} \right] = \rho_o Hg$$

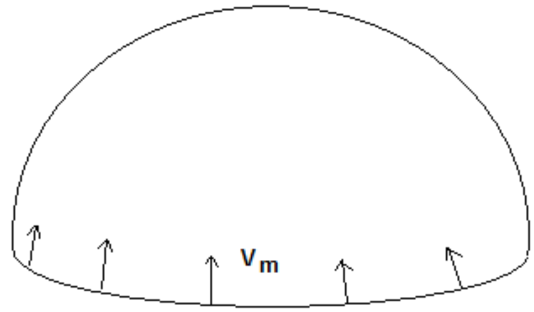
d) plug in values to solve for H:

$$H(m) = \frac{P}{\rho_o g} = \frac{100000(kg/m/s^2)}{1.25(kg/m^3)*9.81(m/s^2)} = 8000(m)$$

e) final answer is **~8000 meters** for the scale height consistent with global mean surface pressure and density.

➔ This value is near to what was assumed in the readings and is the value commonly referred to as the scale height for the atmosphere. Although we assumed a simple exponential relationship for density distribution and global averaged values for the surface variables, we can hypothesize that the scale height does not differ greatly from this value across the globe considering that the extreme ranges of sea-level surface pressure are only about 5% from the global average (considering a 950 mb low in a hurricane and 1050mb arctic high). A 5% variation in scale height is within the rounding-error magnitude for the scale height estimate with the 1 significant figure considered. Deviations in surface air density or air density profiles are not as easy to estimate intuitively as surface pressure and therefore, may deviate enough from our assumptions to cause greater than 5-10% deviations in scale height. One might assume that this may happen in extreme conditions, such as within a very dense arctic air mass, or highly stratified column with various strong inversions.

1.21: Calculate v_m:



- surface area of N. hemisphere: $2.55*10^{14}$ m^2
- change in mass per unit area, M, per month:

$$P_s = \int_0^\infty \rho g dz = Mg$$

$$\frac{dP_s}{dt} = g\frac{dM}{dt}$$

$$100\left(\frac{kg}{ms^2 month}\right) = 9.81\left(\frac{m}{s^2}\right)\left(\frac{dm}{dt}\right)$$

$$\left(\frac{dm}{dt}\right) = 10.2\left[\frac{kg}{m^2 month}\right]$$

- change in mass over entire hemisphere per month, and therefore flux of mass across the equator in one month and one second, M_{ef}:

$$\frac{dM_{nh}}{dt} = \frac{dM}{dt} * A_{nh} = 10.2\left(\frac{kg}{m^2 month}\right) * 2.55*10^{14}\left(m^2\right) = 2.6*10^{15}\left(\frac{kg}{month}\right)$$

$$2.6*10^{15}(kg/month)/2.592*10^6(\text{sec}onds/month) = 10^9(kg/\text{sec})$$

- average mass in a column (per m^2):

$$M_c = \frac{P_s}{g} = \frac{100000(kg/m/s^2)}{9.81(m/s^2)} = 10000(kg/m^2)$$

- length of the equator, L_{eq}:

$$2\pi r_e = 2\pi * 6.37*10^6 m = 4*10^7 m$$

- mass averaged northward velocity across equator:

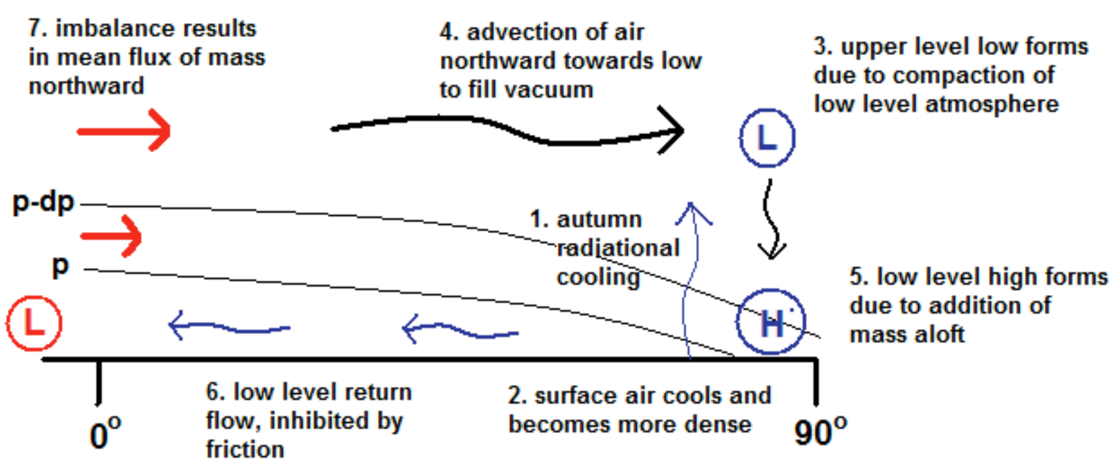
$$v_m = \frac{M_{ef}}{M_c L_{eq}} = \frac{10^9(kg/\text{sec}ond)}{10^4(kg/m^2)*4*10^7(m)} = 0.0025(m/s)$$

- Final answer is **0.0025 m/s**

- *Interpretation:* It is observed that mean surface pressure increases in the northern hemisphere during the transition to winter. This is likely due to the cooling of surface air as the daily insolation decreases in the autumn period. The increasing density of the air at the surface due to the cooling will result in lowering pressure heights aloft. The upper level pressure gradient formed will encourage a flow of air towards this upper level "vacuum" which will increase the mass in the column leading to higher pressure at the surface. This buildup of mass in northern hemisphere will on average require a transfer of mass from the southern to the northern hemisphere at the rate I calculated.

A return of air at low levels will result from the northerly highs to (likely) surface lows caused by the loss of upper air mass in the south. However, the return jet must not occur at the same rate as the upper air transfer, or there would be no average transfer of mass across the equator. The surface return is intuitively slower due to the high friction near to the surface. Air at the top of the atmosphere can likely be advected easier and faster because of low friction due to its lower density and its detachment from the surface friction. In other words, the timescale of response of the upper level advection to the upper level low caused by surface cooling is greater than the timescale of the surface return branch. This imbalance results in an average flux of mass towards the northern hemisphere as mass builds up in the cold dense surface highs and slowly returns towards southerly surface lows.

A simplified diagram of this interpretation is provided below:



Relative cold in the northern hemisphere and hot in the southern hemisphere squirts air northward.