J.D. McAlpine
ATMS 611
Hmwk #3

#3.18

a) The air within the balloon must be less dense than the surrounding air for it to be buoyant. Since density decreases exponentially with height, the air within the balloon must be less dense at a higher altitude than at lower altitude to remain buoyant. Given the ideal gas equation, $\rho=P/RT$, a higher T will give the air within the balloon less density. Given the same temperature, mass, and pressure as a balloon at lower altitude, a bigger balloon could still by buoyant due to its higher volume: given the same mass over a larger volume (due to a light gas such as hydrogen or helium since according to Avogadro's hypothesis that gases containing the same number of molecules occupy the same volume under constant pressure and temperature) the balloon could remain buoyant.
b) The fuel is required to continue heating the air inside of the balloon to make it less dense than the surrounding air. If the balloon is rising through a column of air with a steep lapse rate, than the air will have a steeper lapse rate of density compared to that of an inversion. Therefore, the balloon will have to decrease its density to maintain buoyancy at less of a rate rising through the steep lapse rate column. The same concepts apply in the cold vs. warm day operation: the lower density of the warm air will require more fuel to maintain the lower density of the balloon to maintain buoyancy.
c) At higher altitudes and warmer days, the air is less dense. The airfoil of the plane will operate with less lift at the same airspeed due to the lower amount of mass flowing over the wing. Therefore, the airfoil of a plane must be impacted by a higher velocity of air to create the same pressure difference across the wing to create enough lift for takeoff.
d) Considering Avogadro's hypothesis (gases containing the same number of molecules occupy the same volume of air at the same pressure and temperature), we can visualize this using the ideal gas law: $R \propto V/m$. In the case that you have a kg of moist air and dry air, the volume will be greater for the moist air because a portion of the volume will contain lighter H_2O molecules (18 kg/kmole) than the dry air (28 kg/kmole). Therefore, to have the same mass there must be *more* of the moist air, and therefore a higher volume of the moist air. With greater volume, the gas constant will be greater for the moist air in this case.
e) Air is compressible and therefore the density increases with depth. With greater depth, the rate of increase of weight above will increase exponentially due to this compressibility. This results in an exponential increase in pressure with depth. In the oceans, water is quasi-incompressible and therefore the rate of weight increase with depth remains quasi-constant. This results in a linear increase of pressure with depth.

f) 1) record the station atmospheric pressure
   2) refer to a topographic map or GPS unit to obtain elevation (assuming this is OK in this case)
   3) estimate the scale height, H, assuming an isothermal atmosphere based on the virtual temperature measured at the station.
   4) using the exponential equation (W&H 1.8 or 3.31), estimate the sea level pressure

g) It predicts an exponential decrease of pressure with height for hydrostatic, isothermal conditions. This is because it is derived using the hydrostatic balance assumption and uses a scale height in its equation that is based on a mean temperature.

h) The cooler air within the low will result in lower thicknesses from layer to layer compared to the surroundings, based on the hypsometric equation. At higher heights, the geopotential depression can be thought of as a result of the sum of thicknesses of many layers below the height of reference. The sum/superpositions of these layers will result in further depression of geopotential with height.

i) This is due to an inversion in lower layers resulting in cool air at the surface capped by a warm (and therefore thick) upper layer. This is likely common in an upper-level ridge where sinking air results in warm air aloft and a mid-level inversion. The resulting clear skies allow a lot of radiational cooling at night resulting in cool temperatures at the surface.

j) Within the tire the air was at the same temperature as the surrounding air (given that the air has equalized in temperature with its surroundings through conduction), but at a much higher pressure. As the tire air is released into the surroundings, the pressure is much lower and therefore, the air must cool according to the ideal gas law (considering that the density of air does not change by the same fraction).

k) The change of state must occur during a loss or gain of heat. If no work is done than the first law of thermodynamics tells us that the change in the internal energy of the system must balance the added/lost heat. If heat is added, then the internal energy of the system must go up and no change in volume must occur.

l) In a moist parcel, a portion of the gas within the parcel is lighter H_2O which has a higher specific heat than dry air. Therefore, given the same decrease in pressure with a certain given lift, a greater temperature decrease is required by the dry air to obtain the same pressure decrease in an adiabatic system: $c_p dT = dp/\rho$.
   1) $c_{pm} > c_{pd}$
   2) given $\rho_m \sim \rho_d$ and the same dp:
   3) $dT_m < dT_d$

m) The potential temperature should be constant in a well-mixed layer.

n) The saturation vapor pressure in cold air is very low considering the exponential decrease with lower temperature; therefore cold air would be very dry to begin with (even if near saturation). When this air is mixed into the indoor air and warmed to room temperature it would have a very low relative humidity.

o) This would be a function of the amount of water vapor in the air. The temperature of the sea is much greater off of the coasts of eastern Asia and the eastern U.S., which results in a greater evaporation rate of water into the air and greater vapor pressure.

p) Considering the exponential increase of saturation vapor pressure with temperature, higher temperatures will require an exponentially higher amount of water vapor to achieve high dewpoints. According to Fig. 3.9 (W&H), a temp of ~32° C (90° F) has a saturation vapor pressure of ~50 hPa, which is extremely high and difficult to achieve without a large plane of very warm water present.

q) Human body temperature is regulated through sweat which cools the body through evaporation. With warm humid air, there is a high amount of water vapor in the air and thus it is more difficult to evaporate water in these circumstances.

r) assuming no water vapor phase change (unsaturated through the entire lift):
- saturation mixing ratio will change as the parcel is lifted because the saturation vapor pressure will lower with decreasing pressure.
- therefore: potential temp. and mixing ratio is the proper pair
- however, if there is a phase change than mixing ratio and potential temp. will change as water vapor is condensed out, lowering the amount of water vapor in the air and releasing latent heat into the parcel.

s) potential temp: not conserved because latent heat is released as vapor condenses
equiv potential temp: conserved because latent heat release is accounted for
mixing ratio: not conserved because vapor will condense out, leaving a lower mixing ratio
saturation mixing ratio: not conserved because vapor will condense out

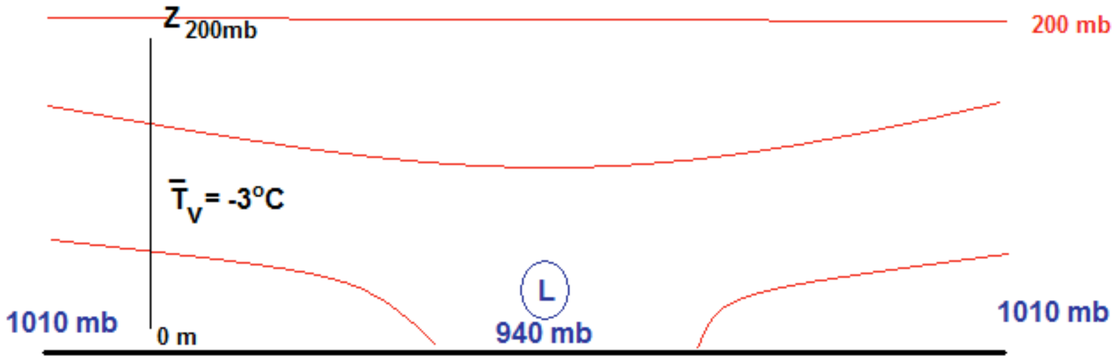
t) The rate of evaporation from a plane of ice is less than that over a plane of water, or in other words, the saturation vapor pressure of ice is less than that of water. Therefore, the air reaches a frost point at a warmer temperature than the dew point because the rate of evaporation from ice is lower and therefore the frost point is "easier" to obtain.

u) Fill the gas tumbler with cold water and place it in the air with the thermometer in place. If the air is not too dry, water should condense on the tumbler, releasing latent heat into the water. A good careful record of temperature should reveal the point where water stopped condensing on the glass and began evaporating (the rate of warming should contain a kink at this point). This should be the dewpoint.

v) The non-perfect heat pump/Carnot engine requires energy to transfer heat from the interior of the refrigerator to the exterior. An open door will cause the pump to work harder to try to maintain the temperature in the interior of the fridge, which will continue to pump heat out and create heat through engine friction and electric heating.

w) At this point the liquid begins to change state to a gas causing gas bubbles to form within its interior. The less-dense gas bubbles will rise to the surface of the liquid. The saturation vapor pressure is where the evaporation rate of the liquid equals the condensation rate of the vapor from the air. If it is exceeded, immediate state change occurs as more liquid changes to a gas because the atmospheric pressure is no longer high enough to keep the gas bubbles from forming.

3.26



outside layer:

$$Z_{top} - Z_{bottom} = Z_{top} = \frac{R_d \overline{T_v}}{g} \ln\left(\frac{P_o}{P_{top}}\right) = \frac{287\left(\frac{J}{kgK^\circ}\right)270^\circ K}{9.8\left(\frac{m}{s^2}\right)} \ln\left(\frac{1010hPa}{200hPa}\right) = 12{,}800 \text{ m}$$

- thickness is same for both the surrounding and interior layers with current assumptions at 12,800 meters:

inside layer:

$$12{,}800m = \frac{R_d \overline{T_v}}{g} \ln\left(\frac{P_o}{P_{top}}\right) = \frac{287\left(\frac{J}{kgK^\circ}\right)\overline{T_v}}{9.8\left(\frac{m}{s^2}\right)} \ln\left(\frac{940hPa}{200hPa}\right)$$

$$\overline{T_v}(inside) = 282^\circ K$$

$$\Delta \overline{T_v} = \overline{T_v}(inside) - \overline{T_v}(outside) = 282^\circ K - 270^\circ K = 12^\circ K$$

The average temperature difference between the interior and exterior is 12° K. Therefore, the center of the hurricane is a big warm bubble.