J.D. McAlpine
ATMS 611
HMWK #5

3.43 Raindrop evaporating (is 12°C) : 18°C air, calculate mixing ratio of air

- sat. mixing ratio at 12°C is 8.7 g/kg (assumed wet-bulb temperature)
- $L_e = 2.25*10^6$ J/kg
- c_{pw}=1952 J/kg/°K
- c_p= 1004 J/kg/°K

- We can assume that the air immediately at the surface of the raindrop is cooled to saturation and therefore the air here is also 12°C and mixing ratio is 8.7 g/kg. In other words, we are assuming that 12°C is the wet-bulb temp. Therefore, our task is to find how many g/kg of water must have been evaporated to cool the air 6°C. We can subtract this amount from the 8.7 g/kg saturation mixing ratio to find the actual mixing ratio in the ambient air.

$$\Rightarrow \quad 6\left[{}^\circ K\right]*1004\left[\frac{J}{kg_{air}{}^\circ K}\right] = 6024\left[\frac{J}{kg_{air}}\right]$$

$$6024\left[\frac{J}{kg_{air}}\right]*\frac{1}{2.25*10^6}\left[\frac{kg_{H2O}}{J}\right] = 0.00268\left[\frac{kg_{H2O}}{kg_{air}}\right]$$

➔ 8.7 g/kg – 2.7 g/kg = ***6.0 g/kg***

This problem demonstrates Normand's rule: water evaporated into the air will cool until the air immediately at the surface of the water (which is also cooled because it is the same temperature as the water) reaches its saturation mixing ratio. This method can be used with a sling psychrometer to determine the ambient mixing ratio.

The problem is also solved using a Skew-T chart (attached) and Normand's rule.

3.45

- 1000 mb, 25°C, saturated adiabatic lapse rate
- how much temp change at 250 mb with 1°C surface temp increase

Refer to attached Skew-T plot
➔ follow saturated adiabatic lapse rates in both cases
➔ plot reveals initial 250 mb temp of ~ -35°C
➔ plot reveals altered temp of ~ +3°C with surface increase of +1°C

3.46

- air parcel: 1000 mb, 15°C, 4°C T_d: use the skew T plot:
(refer to attached skew T plot)
a) - mixing ratio: 5 g/kg
  - relative humidity: 5(g/kg) / 10.5 (g/kg)= 48%
  - wet-bulb temp: 9°C
  - potential temp: 288°K
  - wet-bulb potential temp: 282°K
b) same parameters as for a: but now if parcel rises to 900 mb
  - temperature: about 7°K
  - mixing ratio: 5 g/kg
  - relative humidity: 5(g/kg)/ 6.5 (g/kg)= 77%
  - wet-bulb temp: 4.5°C
  - potential temp: 288°K
  - wet-bulb potential temp: 282°K
c) same parameters as for a: but now if parcel rises to 800 mb
  - temperature: about -1°C
  - mixing ratio: 4.2 g/kg
  - relative humidity: 4.2 g/kg / 4.2 g/kg = 100%
  - wet-bulb temp: 272° K (saturated air mass)
  - potential temp: 293°K
  - wet-bulb potential temp: 282°K
d) lifting condensation level: about 840 mb

3.53

==> refer to attached chart
a) AB: unstable: lifted parcels will be warmer than environment
  BC: neutral: lifted parcels will be same temp. as environment
  CD: neutral: lifted parcels (saturated) will be same temp. as environment
  DE: stable: lifted parcels will be cooler than environment
  EF: stable: lifted parcels will be cooler than environment
  FG: slightly stable: lifted parcels will be slightly cooler than environment
b) convectively unstable?
  AB: yes
  BC: yes
  CD: no, neutral
  DE: yes, but a lot of energy is required to push parcels up to the LFC since this layer contains a strong inversion
  EF: yes, but a decent amount of energy is required to saturate the layer
  FG: yes

3.55

Show that the conditions for the formation of a mirage (the increase of density with height) are realized if the decrease in atmospheric temperature with height exceeds $3.5*\Gamma_d$ (ignored the suggestion about the D equation, since the problem can seemingly be solved by just using the ideal gas equation)

➔

$$p = \rho R_d T$$

$$\ln P = \ln \rho + \ln R_d + \ln T$$

$$\frac{1}{p}\frac{dp}{dz} = \frac{1}{\rho}\frac{d\rho}{dz} + \frac{1}{T}\frac{dT}{dz}$$

$$\frac{1}{\rho}\frac{d\rho}{dz} = -\frac{1}{T}\frac{dT}{dz} + \frac{1}{p}\frac{dp}{dz}$$

$$\frac{1}{\rho}\frac{d\rho}{dz} = -\frac{1}{T}\frac{dT}{dz} - \frac{\rho g}{p}$$ (from hydrostatic equation)

$$\frac{1}{\rho}\frac{d\rho}{dz} = -\frac{1}{T}\frac{dT}{dz} - \frac{g}{R_d T}$$ (from ideal gas equation)

$$\frac{1}{\rho}\frac{d\rho}{dz} = \frac{-1}{T}\left(\frac{dT}{dz} + \frac{g}{R_d}\right)$$

- ➔ conditions for mirage: $d\rho/dz > 0$: density increases with height
- ➔ assuming $\Gamma_d = g/c_p$, therefore:

$$\frac{1}{\rho}\frac{d\rho}{dz} = \frac{-1}{T}\left(\frac{dT}{dz} + \frac{g}{R_d}\right) > 0$$

$$\left(\frac{dT}{dz} + \frac{g}{R_d}\right) < 0$$

$$\frac{dT}{dz} < -\frac{g}{R_d}$$

$$\frac{dT}{dz} < -\frac{c_p}{R_d}\Gamma_d$$

$$\frac{dT}{dz} < -\frac{1004}{287}\Gamma_d$$

$$\frac{-dT}{dz} > 3.5\Gamma_d$$

therefore, it is shown that for mirage conditions to occur ($d\rho/dz > 0$), the lapse rate dT/dz must exceed 3.5 times the dry adiabatic lapse rate (temperature needs to fall off greater than 34.3 °K/km)

UNR-DRI Lapse rate problem:

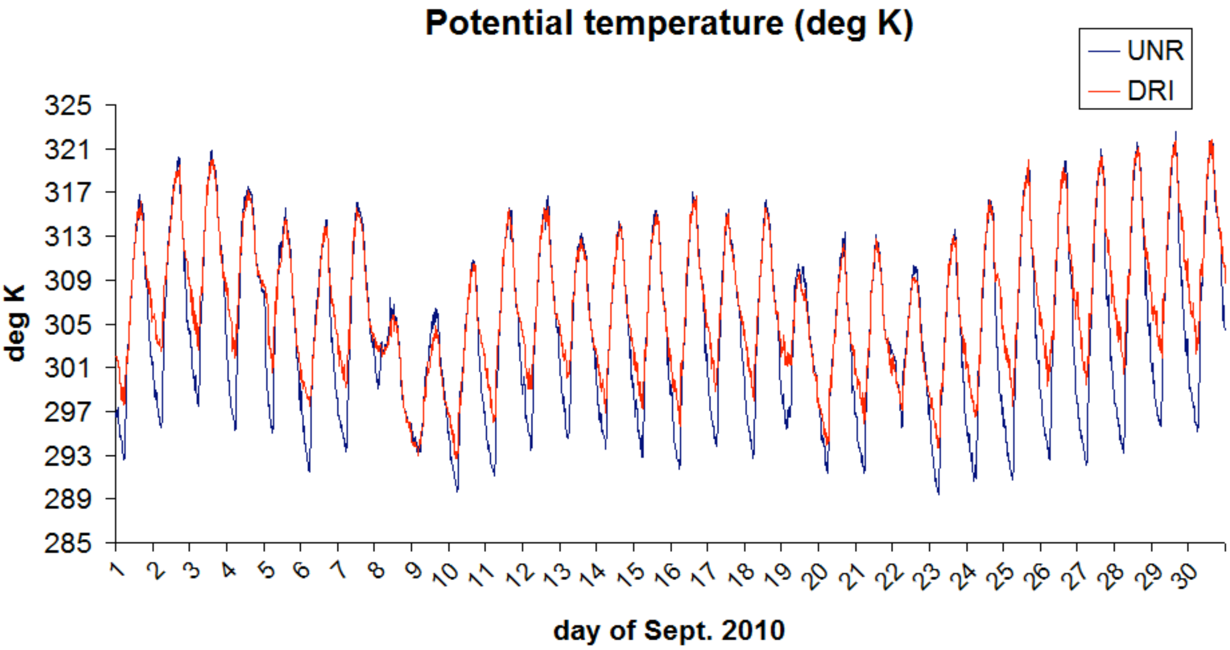
The height difference of two ground based weather stations (UNR and DRI) in Reno is 143 meters. Obtain data from these two sites (from the historical data link: Password is wrcc14) for the month of September 2010.

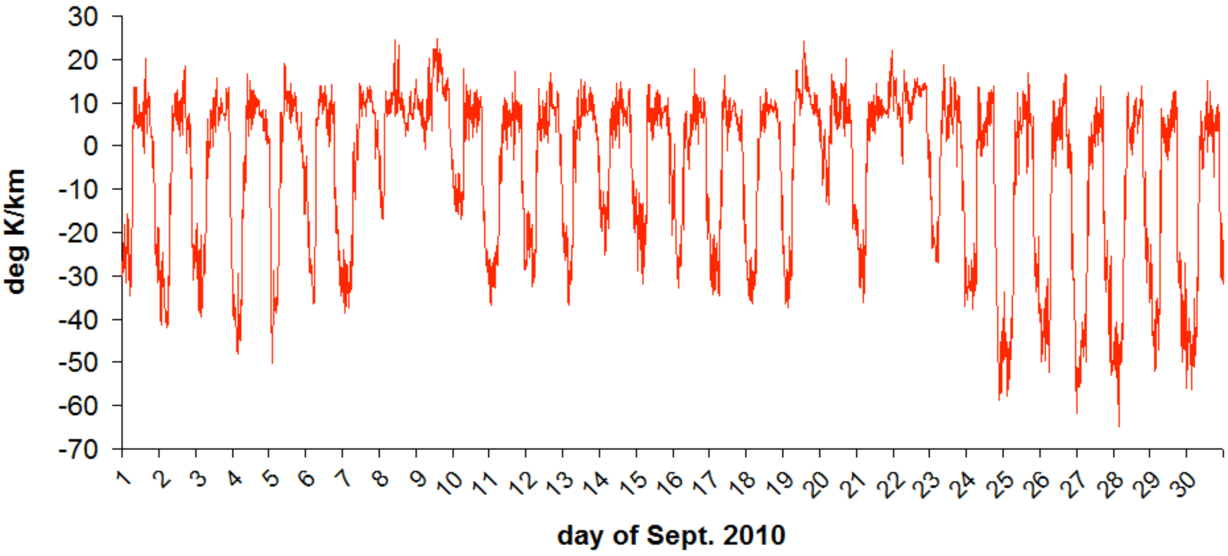
➔ refer to spreadsheet: pressure and temperature downloaded for both stations

a) Calculate and plot the lapse rate and the potential temperature at these two sites as a function of time.

➔ use Poisson eqn to determine potential temp (brought down to 1000 mb):

$$\Theta = T\left(\frac{p_o}{p}\right)^{\frac{R_d}{c_p}}$$




Lapse rate between DRI and UNR (deg K/ km)

b) What fraction of time is there a temperature inversion?

➔ roughly 43% of the time (refer to spreadsheet)
➔ inversion conditions occur almost every night from sundown to sunrise

c) Are there times when the lapse rate is greater than the adiabatic lapse rate?
➔ yes, it is a common occurrence, lapse rate rising >9.8 °K/km almost every day during afternoon hours.

d) Interpret findings:

First of all, taking a look at the potential temperature comparison plot a number of patterns are apparent. First of all, both plotlines reveal the diurnal pattern of warming and cooling corresponding to periods of incoming and outgoing radiation. The DRI station has a smaller average range of minimum-maximum potential temperature spread compared to the UNR station. Both datasets are fairly close in value during warming and cooling periods. However, during the peak temperature hours, the UNR temperature tends to peak above the temperature of the DRI station. During nighttime hours, the UNR potential temperature cools to a much greater degree than the DRI temperature. Thus, a strong inversion occurs almost nightly.

This pattern is indicative of the decoupling of valley air from the hilltop air during night. At night the cool air pools into the valley "bowl", forming a strong inversion which prevents mixing with the air above it. Once the daily solar radiation begins, the valley air quickly warms and begins mixing with the upper level air, resulting in a mixed layer with a near constant potential temperature. This decoupling is not evident in days 8-10, likely due to the influence of a synoptic disturbance. The lack of heating during the day (based on the low maximum potential temps.) suggests that clouds, wind, and possibly precipitation occurred during this period. These conditions promote a well-mixed neutral atmosphere and as a result, the potential temperatures are equivalent at both stations.

The Lapse Rate plot reveals the strength of the inversion that occurs at night. Maximum inversion magnitudes reach nearly -70°K/km: very strong inversions due to the high loss of heat due to the high and dry desert air and the pooling of cooled air in the valley.

The Lapse Rate plot also reveals that the lapse rate becomes super-adiabatic nearly every day during maximum heating hours. As a result, we might expect that thermal plumes occur over the city nearly every day during this period. We can assume that scattered cumulus clouds occurred around the valley during this period, if sufficient moisture was present. The UNR site is also located within an urban area and the extra heating due to lower albedo (compared to the hills) and mechanical activity could be resulting in a localized super-adiabatic region.