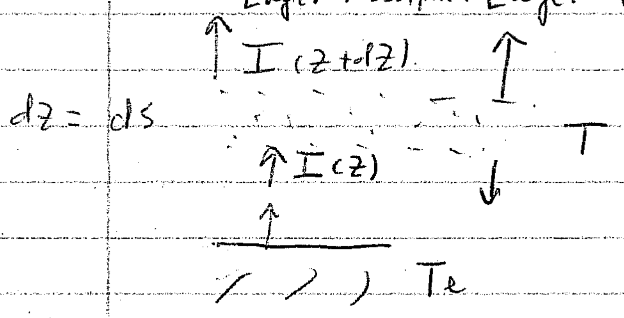
Schwarzchild's Equation

Material from CH8 of Petty, on website



$$B(T,\nu) = P_e(T,\nu)$$

$$B(T,\nu) = \frac{2h\nu^3}{c^2\left(e^{h\nu/kT}-1\right)}$$

Planck Blackbody function

(1) $$I(z+dz) = \underbrace{I(z)}_{\text{What comes in}} - \underbrace{I(z)\,\beta_a\,dz}_{\text{Taken out by absorption}} + B(T(z),\nu)\,\beta_a(\nu)\,dz$$

$\beta_a dz$: Layer Absorptivity

$\beta_a(\nu) dz$: Layer emissivity

Here

$$\beta_a = \sum_{i=1}^{\#\text{ emitting gases}} \sigma_{abs}^{(i)} N_i$$

$\sigma_{abs}^{(i)}$: Absorption cross section per molecule

N_i: # absorbing molecules / Volume.

Regrouping (1),

(2) $$\boxed{\frac{dI}{dz} = \frac{I(z+dz) - I(z)}{dz} = \beta_a\,(B - I)}$$

Schwarzchild's Equation ⇒ Equation of radiation transfer in an absorbing (non scattering) medium.

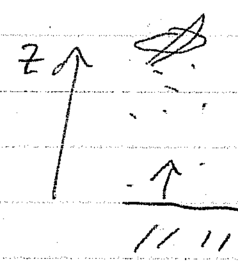
Differential Form of the Equation.

Note: When $B \ll I$ (like solar λ),

Beer-Bougier Equation,

Aircraft at Z wants to know!!

$$dI = -\beta_a I \, dZ$$



$$I(z) = I_o \, e^{-\int_0^Z \beta_a(z')dZ'}$$

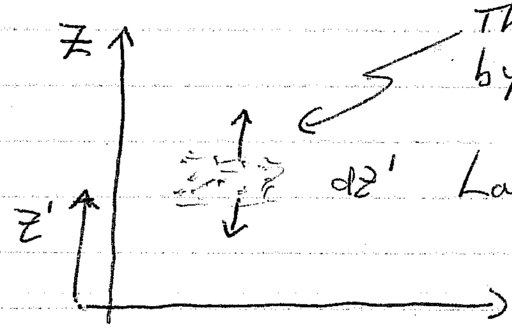
$I_o = I(z=0)$ — some upward radiance,

Te

typically $I_o = B(T_e)$

Atmosphere Emission works the same way:

This emitted radiance is attenuated by atmosphere between z' and Z.



dz' Layer Emission $= B \beta_a dZ'$

Emission by layer dz' that makes it to Z:

$$\beta_a(z')dZ' \, B[T(z')] \, e^{-\int_{z'}^{Z} \beta_a(Z'')dZ''}$$

Let

$$\tau(Z') \equiv \int_{z'}^{Z} \beta_a(Z'')dZ'' , \quad \text{So } \tau(Z) = 0 .$$

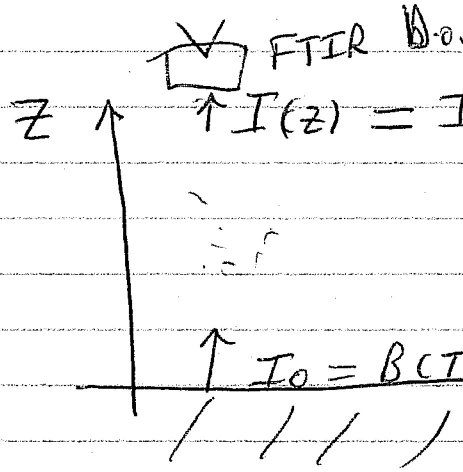
Then total emission by the atmosphere, as attenuated, is

$$\cancel{\underbrace{I(z)}_{\text{Atmosphere emission}} = \int_0^Z \beta_a(z')dz' \, B[T(z')] \, e^{-\tau(z')}}$$

Again, the total emission by the atmosphere, as attenuated, is

$$I_{(z)}^{\text{Atmosphere emission}} = \int_0^z e^{-\tau(z')} B[T(z')]\, \beta_a(z')\, dz'$$

The total is



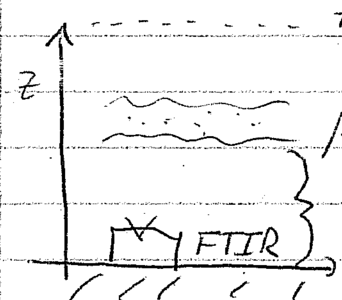
$$(3) \qquad I(z) = I_0 e^{-\tau(0)} + \int_0^z e^{-\tau(z')} B[T(z')]\, \beta_a(z')\, dz'$$

Here $\tau(0) = \int_0^z \beta_a(z')\, dz'$

Absorption optical depth for atmosphere to height z.

The discrete form of this equation is solved in my handout for IR.

(3) is the integral form of the Schwarzchild's Equation.



$\beta_a(z')\, dz'\, B[T(z')]$

$$t(z') \equiv e^{-\int_0^{z'} \beta_a(z')\, dz'}$$

$$I^{\downarrow}(\nu) = \int_0^{\infty} t(z')\, \beta_a(z')\, B[T(z')]\, dz'$$

Upward looking IR Spectrometer, gases only!

Examples Next:

1) Power Point Presentation showing Various Spectra

2) Homework Problem

A) Explain in your own words what figures 8.2 a and b show.

B) On Figure 8.2 B, show regions of the spectrum where major gases contribute to the downward emission [CO_2, H_2O, O_3, CH_4]

C) Identify the atmosphere window region, describe why it is a window, and give the wavenumbers and wavelengths associated with this window.