

# New determination of Rayleigh scattering in the terrestrial atmosphere

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New Rayleigh-scattering optical thickness values for the terrestrial atmosphere in the  $260 < \lambda < 1500$ -nm wavelength range have been calculated using updated data on atmospheric optical parameters. The calculations include molecular scattering from water vapor and take into account varying atmospheric composition with altitude. The new Rayleigh-scattering coefficients average 4.5% lower than those listed by Penndorf in 1955.

## I. Introduction

It has been apparent in the last few years that tabulations of Rayleigh-scattering coefficients need revision. New information on the depolarization factor and on the refractive indices for gases impelled us to recalculate the Rayleigh-scattering coefficients: the new coefficients (expressed in terms of optical depth) are presented here for five model atmospheres. We have included Rayleigh scattering from water vapor in the computations to make the tabulations more applicable to realistic atmospheres. At the time we were doing the calculations, Hoyt,<sup>1</sup> using updated data on the depolarization factor, published tables on a redetermination of Rayleigh-scattering optical thickness. His results were 3.5% lower than those listed by Elterman<sup>2</sup> or Penndorf,<sup>3</sup> while our results average 4.5% lower. The 1% discrepancy between Hoyt's values and ours is due to (1) inclusion of water vapor scattering in the present listings, (2) inclusion of varying oxygen-nitrogen ratios with altitude, and (3) slightly different values of refractive index of  $O_2$  and  $N_2$ .

## II. Theory and Error Analysis of the Rayleigh Optical Depth

The Rayleigh extinction coefficient  $k_R$  is calculated for anisotropic molecules according to the following formula<sup>4</sup>:

$$k_R = 24 \frac{\pi^3}{\lambda^4} \cdot \frac{1}{N} \left( \frac{n^2 - 1}{n^2 + 1} \right)^2 \frac{6 + 3\delta}{6 - 7\delta}, \quad (1)$$

where  $k_R$  is the Rayleigh-scattering extinction coefficient ( $m^{-1}$ ),  $\lambda$  is the wavelength (m),  $N$  is the number density of molecules ( $m^{-3}$ ),  $n$  is the refractive index of air, and  $\delta$  is the depolarization factor,  $(6 + 3\delta)/(6 - 7\delta)$ , denoted hereafter as  $D$ . In the atmosphere both the refractive index  $n$  and the molecular number density  $N$  depend on pressure and temperature and hence on elevation. According to the Lorenz-Lorentz equation, the term  $(n^2 - 1)/(n^2 + 1)$  is proportional to  $N$ , and therefore the following expression is normally substituted in the formula of  $k_R$ :

$$\frac{(n^2 - 1)^2}{N} = \frac{(n_s^2 - 1)^2}{N_s} \cdot \frac{N}{N_s}, \quad (2)$$

where  $n_s$  is the refractive index of air at standard conditions (15°C, 760 Torr). This substitution can only be done if the influence of humidity on the refractive index is ignored. In our program, we included the water vapor and used for the pressure, temperature, and humidity dependence the expression given by Edlén<sup>5</sup>:

$$n_{tpf} = 1 + (n_s - 1) \cdot \frac{0.00138823 \cdot p}{1 + 0.003671 \cdot t} - f \cdot (5.722 - 0.0457\sigma^2)10^{-8}, \quad (3)$$

where  $n_{tpf}$  is the refractive index of moist air,  $n_s$  is the standard refractive index of air with 330 ppm  $CO_2$ ,  $p$  and  $t$  are the pressure (Torr) and temperature (°C),  $f$  is the water vapor pressure (Torr), and  $\sigma$  is the vacuum wave number ( $\mu m^{-1}$ ). For the calculation of  $n_s$ , we use a formula proposed by Peck and Reeder.<sup>6</sup> This formula represents the refractive index of air in the near IR more accurately than Edlén's formula. It has the following form:

$$(n_s - 1) \cdot 10^8 = \frac{5791817}{238.0185 - \sigma^2} + \frac{167909}{57.362 - \sigma^2}. \quad (4)$$

The rms deviation from the above formula from the experimental data is  $< 0.4 \times 10^{-8}$  for the 0.23–2.0- $\mu m$  spectral range.

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Unfortunately, no results from a direct determination of the depolarization factor of air are available. Therefore, we had to calculate it from other data. Experimental results exist from the main constituents of the atmosphere, i.e., oxygen, nitrogen, and argon. To combine these data we use the method proposed by Kasten<sup>7</sup>:

$$(n_{\text{air}}^2 - 1)^2 \cdot D_{\text{air}} = \sum_i (n_i^2 - 1)^2 \cdot D_i \cdot C_i, \quad (5)$$

where index  $i$  represents different constituents of the atmosphere and  $C_i$  their concentration. This formula is based on the assumption of incoherent scattering. It is, therefore, appropriate to simply sum the cross sections of the individual scattering terms. The  $D_i$  or  $\delta_i$  have been measured for oxygen, nitrogen, and argon by Weber *et al.*<sup>8</sup> and Rudder and Bach.<sup>9</sup> Their values for unpolarized radiation are listed in Table I with the refractive indices and relative concentrations in the atmosphere at 587.9 nm, 0°C, and 760 Torr. We used the

parameters of argon to represent all the atmospheric gases other than oxygen and nitrogen; this is acceptable because argon is the dominant rare gas. For the calculation of  $D_{\text{air}}$  we assume that the depolarization factors are wavelength independent. The refractive index of air at 587.9 nm, 0°C, and 760 Torr is calculated with the above formulas and yields  $n_{\text{air}} = 1.00029240$ . With these data, the following results are obtained:

$$D_{\text{air}} = 1.0160 \pm 0.0068, \quad \delta_{\text{air}} = 0.0095 \pm 0.0040.$$

The  $\delta_{\text{air}}$  value is  $\sim 1.5$  times lower than the one calculated by Hoyt.<sup>1</sup> The accuracy of the calculated  $D_{\text{air}}$  is mainly determined by the accuracy of the refractive indices of nitrogen and oxygen, which is only  $\pm 1.10^{-6}$ . The accuracy of the Rayleigh optical depth due to this uncertainty is  $\pm 0.7\%$ , or in absolute values of optical thickness the accuracies are  $\delta\tau = \pm 0.08$  at  $\lambda = 300$  nm and  $\delta\tau = 0.001$  at  $\lambda = 500$  nm. ( $\tau$  is the vertical optical thickness of the atmosphere.) As other uncertainties due to number density variation and changes in atmospheric composition are  $< 0.1\%$  level, they can be ignored as long as no more accurate depolarization factors are available. At the moment, this accuracy seems to be sufficient.

Sun photometer data taken from high altitude stations, such as at the Mauna Loa Observatory, show aerosol optical thicknesses at 500 nm of the order of the difference value stated.<sup>10</sup> Therefore, it is thought that this difference is also responsible for the so-called anomalous or negative extinction of aerosols (super-Rayleigh atmosphere) sometimes reported. For the influence on other remote sensing problems reference is made to Hoyt's paper.<sup>1</sup>

Table I. Optical Parameters and Relative Concentrations of Atmospheric Gases

	$\delta \times 10^2$	$D$	$n$	Concentration
Nitrogen (Ref. 9)	$1.17 \pm 0.06$	$1.0198 \pm 0.0010$	1.000297	0.781
Oxygen (Ref. 8)	$1.4 \pm 0.1$	$1.0237 \pm 0.0017$	1.000272	0.209
Argon (Ref. 9)	$0.002 \pm 0.001$	$1.00003 \pm 0.00002$	1.000281	0.010

Table II. Rayleigh Optical Depth for Airmass 1, Reduced to 1013 mb<sup>a</sup>

Wavelength (nm)	Atmospheric models				
	15°N, mean	45°N, winter	45°N, summer	60°W, winter	60°N, summer
260.0	2.18505	2.18313	2.18522	2.18196	2.17904
280.0	1.57963	1.57832	1.57979	1.57748	1.57533
300.0	1.17273	1.17180	1.17286	1.17118	1.16957
320.0	0.89022	0.88953	0.89032	0.88907	0.88783
340.0	0.68867	0.68816	0.68876	0.68780	0.68683
360.0	0.54153	0.54114	0.54161	0.54086	0.54010
380.0	0.43196	0.43166	0.43202	0.43143	0.43082
400.0	0.34894	0.34869	0.34899	0.34851	0.34802
420.0	0.28505	0.28486	0.28509	0.28471	0.28430
440.0	0.23522	0.23506	0.23525	0.23494	0.23460
460.0	0.19587	0.19574	0.19590	0.19564	0.19536
480.0	0.16445	0.16434	0.16448	0.16426	0.16402
500.0	0.13911	0.13902	0.13913	0.13895	0.13875
550.0	0.09424	0.09418	0.09426	0.09413	0.09400
600.0	0.06613	0.06609	0.06614	0.06606	0.06596
650.0	0.04778	0.04775	0.04779	0.04773	0.04766
700.0	0.03539	0.03537	0.03540	0.03535	0.03530
750.0	0.02678	0.02676	0.02678	0.02675	0.02671
800.0	0.02063	0.02062	0.02064	0.02061	0.02058
850.0	0.01616	0.01615	0.01616	0.01614	0.01611
900.0	0.01283	0.01282	0.01283	0.01282	0.01280
1000.0	0.00840	0.00839	0.00840	0.00839	0.00838
1100.0	0.00572	0.00572	0.00572	0.00572	0.00571
1200.0	0.00404	0.00403	0.00404	0.00403	0.00402
1300.0	0.00293	0.00292	0.00293	0.00292	0.00292
1400.0	0.00217	0.00217	0.00217	0.00217	0.00217
1500.0	0.00165	0.00165	0.00165	0.00165	0.00164

<sup>a</sup> Elevation: 0.00 km.

### III. Computations and Results

For the computation of the molecular scattering optical depth, the following U.S. standard atmospheres were used<sup>11</sup>: 15°N annual; 45°N, Jan.; 45°N, July; 60°N, Jan.; 60°N, July. The standard atmospheres have been supplemented by the humidity and ozone data used by McClatchey *et al.*<sup>12</sup>

To evaluate the dependence of the optical depth on elevation, to see if it scaled differently from air pressure, calculations were made and are illustrated for 0–4000-m elevations. Furthermore, the total amount of water vapor in the vertical column was varied, but its influence was found to be <0.1%.

The results of the optical thickness calculations as a function of wavelength, model parameters and elevation are shown in Table II. From these data, analytical formulas of the form

$$\tau_R(\lambda) = A \cdot \lambda^{-(B+C\lambda+D/\lambda)} \quad (6)$$

have been derived by least squares curve fitting to the calculated data. The constants  $B$ ,  $C$ , and  $D$  are model and elevation independent and have the values

$$B = 3.916, \quad C = 0.074, \quad D = 0.050.$$

The constant  $A$ , however, is pressure and hence elevation and model dependent. The following expression for  $A$  was determined from the results of the curve fitting:

$$A = (p/p_0)(\alpha + \beta \cdot H), \quad (7)$$

where  $p$  is the station pressure,  $p_0 = 1013.26$  mb,  $H$  is the elevation in kilometers, and  $\alpha, \beta$  are model-dependent constants shown in Table III. The accuracy of the optical depth calculated by this formula depends on wavelength as shown in Table IV. In the 300–900-nm range, the maximum error is less than the uncertainty due to the known accuracy of the depolarization factor. In contrast to this type of formula, the maximum error of the simple Penndorf type formula,

$$\tau_R(\lambda) = A \cdot \lambda^{-B}, \quad (8)$$

is much larger. In the 300–900-nm range it ranges from +2.5 to –1.2%, which is intolerable in view of the  $\pm 0.7\%$  accuracy of the Rayleigh optical depth itself.

In general, the results show that optical depths change only slightly from one atmospheric model to another so that the choice of the model is not very critical. The maximum difference is found at 4-km elevation between 45°N summer and 60°N winter with a 0.35% difference. The height correction also introduces changes of only a few tenths of 1%. For most applications the formula at the end of this paper is accurate enough to estimate the vertical optical depth.

### IV. Conclusions

The redetermination of the vertical optical depth of Rayleigh or molecular scattering shows that the classical values given by Penndorf<sup>3</sup> and Elterman<sup>2</sup> overestimate the optical depth by  $\sim 4.5\%$ . The uncertainty of the present calculations ( $\pm 0.7\%$ ) is mainly due to the lack of accurate depolarization data. It is proposed that

Table III. Constants  $\alpha$  and  $\beta$  for Model Atmospheres<sup>a</sup>

Model	$\alpha \times 10^3$	$\beta \times 10^3$
15°N, annual	8.3829	0.0065
45°N, winter	8.3781	0.0036
45°N, summer	8.3827	0.0069
60°N, winter	8.3715	0.0027
60°N, summer	8.3617	0.0047

<sup>a</sup> See Eq. (7).

Table IV. Maximum Error in Computation Using Eq. (7)

Wavelength range (nm)	Maximum error (%)
250–300	0.8
300–900	0.1
900–1200	0.7
1200–2000	5.4

either a direct determination of the depolarization of air be performed or that the refractive indices of  $N_2$  and  $O_2$  be redetermined to an accuracy of the order of  $\pm 1 \times 10^{-8}$ ; this is possible with modern interferometers used in meteorology.

Until better data are available, the following formula can be used to calculate the Rayleigh optical depth:

$$\tau_R(\lambda) = 0.00838 \cdot \lambda^{-(3.916+0.074\lambda+0.050/\lambda)} \quad (9)$$

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