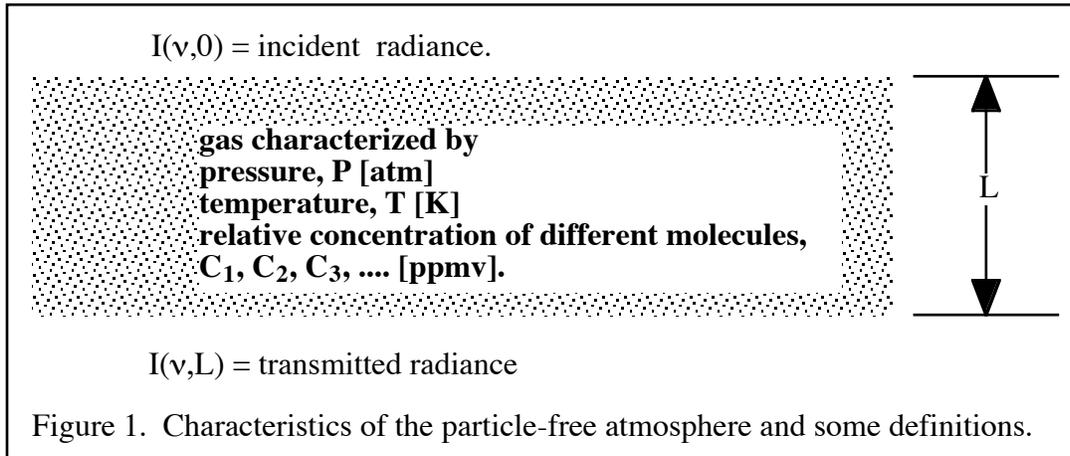


Approximate IR Emission and Transmission by Gases in the Atmosphere

by Pat Arnott

1. Single layer absorption and transmission.

Consider a layer of particle-free atmosphere (i.e. a collection of gas molecules, N_2 , O_2 , CO_2 , Ar, H_2O , O_3 , CH_4 , etc; no clouds or aerosol).



The transmission coefficient $T(v,L)$ is defined as, and computed from;

$$T(v,L) \equiv \frac{I(v,0)}{I(v,L)} = e^{-\tau_{\text{abs}}(v,L)} = e^{-\alpha(v)L} \quad (1)$$

where ν is the wavenumber [cm^{-1}], $\alpha(\nu)$ [cm^{-1}] is the absorption coefficient, and L [cm] is the path length. The absorption coefficient is obtained from a sum over the different molecules as

$$\alpha(\nu) = \alpha_1(\nu) + \alpha_2(\nu) + \dots = \sum_{i=1}^{\text{\# absorbing molecules}} \alpha_i(\nu) \quad (2)$$

The absorption coefficient for molecules of type 1 is obtained from the product of absorption efficiency $B_{\text{abs}1}(\nu)$ [$\text{cm}^2/\text{molecule}$] (i.e. how 'strong' each molecule absorbs) and the molecular density N_1/V [$\text{molecules}/\text{cm}^3$] (i.e. how many molecules of this type per unit volume). The absorption coefficient for molecules of type 1 can be defined as, and by using the ideal gas

'law' plus the related 'law' of partial pressure, expressed as

$$\begin{aligned}\alpha_1(\nu) &= B_{\text{abs}1}(\nu) \left[\frac{\text{cm}^2}{\text{molecule}} \right] \frac{N_1 \left[\frac{\text{molecules}}{\text{cm}^3} \right]}{V} , \\ &= B_{\text{abs}1}(\nu) \left[\frac{\text{cm}^2}{\text{molecule}} \right] \frac{7.244 \times 10^{15} C_1 [\text{ppmv}] P [\text{atm}]}{T [\text{K}]} ,\end{aligned}\quad (3)$$

where the variables are defined in Fig. 1. Note that the relative concentration C_1 is related to the number of molecules of type 1 and the total number of molecules, and the partial pressure of gas 1 and the total pressure as

$$C_1 [\text{ppmv}] = 10^6 \frac{N_1}{N} = 10^6 \frac{P_1 [\text{atm}]}{P [\text{atm}]} . \quad (4)$$

The absorption efficiency $B_{\text{abs}1}(\nu)$ will be computed using the HITRANPC code.

By the way, the ideal gas law is used to obtain N/V , the number of molecules per unit volume, and to relate it to pressure and temperature. The total pressure of a collection of N point-like molecules per unit volume, having average kinetic energy kT per molecule, where k is the Boltzmann constant $k=1.3805 \times 10^{-23}$ [J/(molecule K)], is related to molecular density as

$$\begin{aligned}\frac{P}{kT} &= \frac{N}{V} \\ &= \frac{N_1}{V} + \frac{N_2}{V} + \frac{N_3}{V} + \dots \\ &= \frac{P_1}{kT} + \frac{P_2}{kT} + \frac{P_3}{kT} + \dots\end{aligned}\quad (5)$$

The last form of the equation is the "law of partial pressure", that the total pressure is the sum of the contribution to the pressure by molecules of type 1, of type 2, etc. The second form of the equation simply breaks down the number of molecules into those of the various types, and is the basis for the law of partial pressures. It is implicit in this simple equation of state that molecules occupy no volume of space (V has no correction for the real available volume), and that only

molecular kinetic energy is included, no molecular attraction or repulsion terms. It is a simple equation of state, and is largely reasonable for dilute gas mixtures.

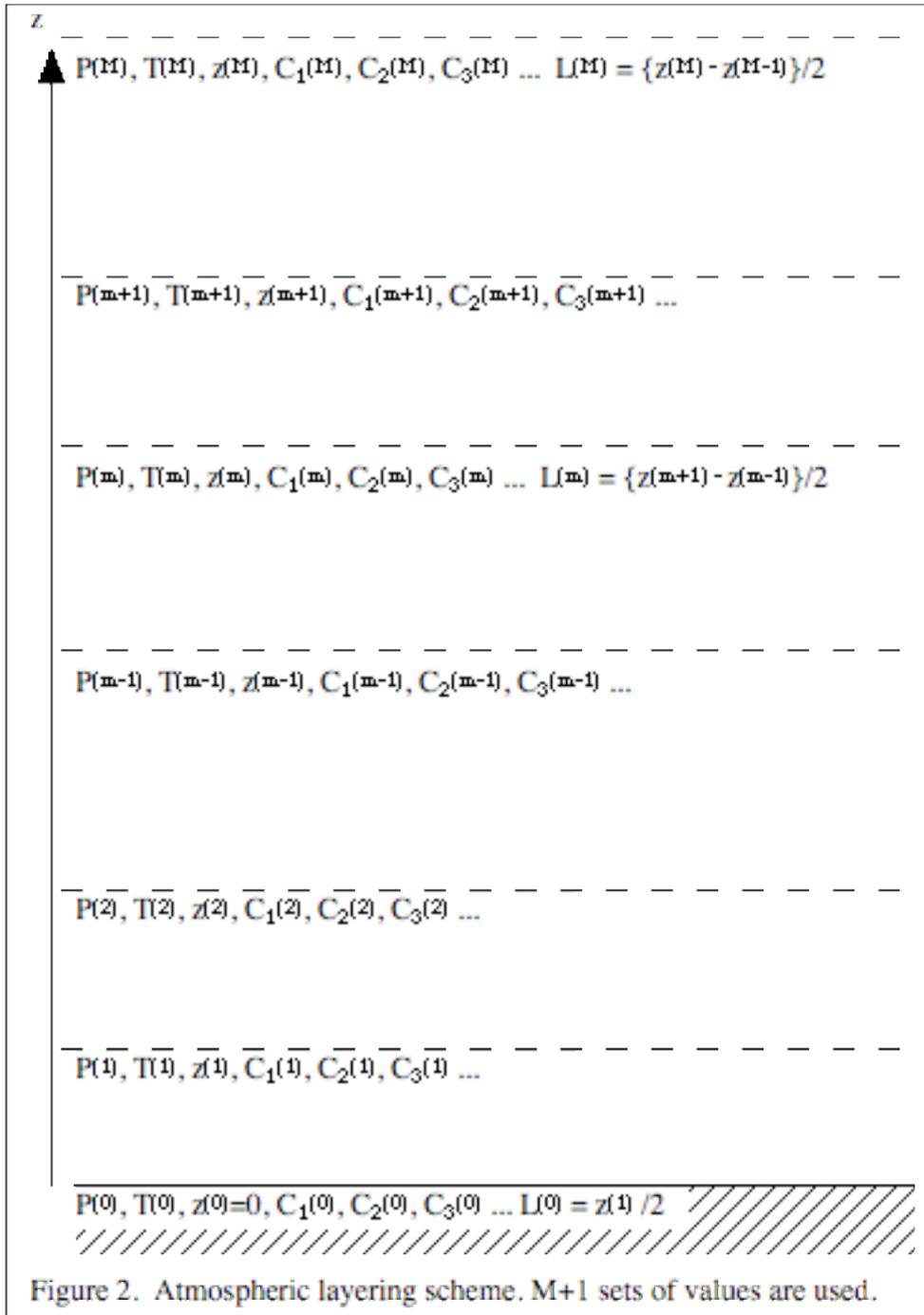
One may obtain the pressure, P, and temperature, T, from a sounding. The concentration from the well-mixed gases CO₂ and CH₄ are easy to obtain, though the concentration of O₃ must be obtained from knowledge of the stratosphere. To obtain the partial pressure of H₂O, the following can be used:

```
* Function to compute vapor pressure at sat, From Liou,  
* pg 451, "Radiation and cloud processes..."  
* VaporPressLiou is in [atm] units.  
* Tkel is the dew point temperature in Kelvin.  
  REAL FUNCTION VaporPressLiou(Tkel)  
  IMPLICIT REAL (a-h,o-z)  
  T0=273.15  
  IF (Tkel.LE.T0) THEN  
    a=21.874  
    b=7.66  
  ELSE  
    a=17.269  
    b=35.860  
  END IF  
  VaporPressLiou=6.11*EXP(a*(Tkel-T0)/(Tkel-b))/1000.  
  RETURN  
  END
```

The partial pressure of H₂O can be converted into a relative concentration using Eq. (4).

2. Contribution of emission at the ground by an arbitrary layer.

The layering scheme is shown in Figure 2.



The contribution at the surface (where the FTIR is located) of the downward radiance emitted by the m th atmospheric layer, and partially absorbed by lower layers, is

$$I^{(m)}(\nu) = B(T^{(m)}) \left[\frac{\text{mW}}{\text{m}^2 \text{ Sr cm}^{-1}} \right] \left(1 - e^{-\tau_{\text{abs}}^{(m)}(\nu, L^{(m)})} \right) e^{-\sum_{i=0}^{m-1} \tau_{\text{abs}}^{(i)}(\nu, L^{(i)})} . \quad (6)$$

The total downward radiance is

$$I(\nu) = \sum_{m=0}^M I^{(m)}(\nu) \left[\frac{\text{mW}}{\text{m}^2 \text{ Sr cm}^{-1}} \right] . \quad (7)$$

The expression for blackbody radiance is

$$B(T, \nu) = \frac{10^5 \nu^3 2hc^2}{e^{hc\nu/kT} - 1} \left[\frac{\text{mW}}{\text{m}^2 \text{ Sr cm}^{-1}} \right] , \quad (7)$$

where the wavenumber ν needs to be given in m^{-1} units, h is Planck's constant, k is Boltzmann's constant, and c is the speed of light. These constants all need to enter the equation in MKSA units. The brightness temperature [Kelvin] can be obtained from the downward radiance as

$$T(\nu) = \frac{hc\nu}{k \ln \left(1 + \frac{\nu^3 2hc^2}{I(\nu)} \right)} , \quad (8)$$

where all quantities should be converted to MKSA units prior to the calculation.

The temperature dependence enters through the blackbody function in Eq. (5), through the gas concentration in Eq. (3), and (secondarily) in the line strengths and spectral broadening factors used to obtain the absorption efficiency in Eq. (3). If the absorption efficiency were independent of temperature the computation of downward radiance would be relatively simple.

3. Spectral averages.

To make comparisons with the observed spectra, the results of this high resolution calculation are averaged using a triangular window function to the spectral resolution of the spectrometer during measurements, e.g. 1 cm^{-1} . The triangular window function $\text{WN}(\nu; \nu_c, \Delta\nu)$ is given by

$$\text{WN}(\nu; \nu_c, \Delta\nu) = 1 - \frac{|\nu - \nu_c|}{\Delta\nu}, \quad (\nu_c - \Delta\nu) \leq \nu \leq (\nu_c + \Delta\nu) \quad , \quad (9)$$

where ν_c is the wavenumber where the emission is desired, and $\Delta\nu$ is the desired spectral resolution. The spectrally averaged emission spectra $I_{spec}(\nu_c)$ is obtained from

$$I_{spec}(\nu_c) = \frac{\sum_k \text{WN}(\nu_k; \nu_c, \Delta\nu) I_{spec}(\nu_k)}{\sum_k \text{WN}(\nu_k; \nu_c, \Delta\nu)} \quad , \quad (10)$$

and can be compared with measurements.

A section of code that implements spectral smoothing is below:

```

* Spectral smoothing subroutine.
* INPUTS:
* del=desired spectral resolution.
* wn = array containing wavenumbers used in high resolution calculation.
* wnaive = array of wavenumbers in the averaged spectrum.
* spec = array containing radiance from the high resolution calculation.
* OUTPUT:
* specave = array of radiance values in the averaged spectrum.
* NOTES:
* del, wn, and wnaive should be in the same units, e.g. 1/cm.
* spec and specave will be in the same units.
  SUBROUTINE SpectralSmooth(del,wn,spec, wnaive, specave)
  IMPLICIT REAL (a-h,o-z)
  REAL wn(452940),spec(452940),wnaive(779), specave(779)
* Set your array dimensions to what you use in your problem.
  EXTERNAL win

  DO 30 j=1,779
  Ts=0.
  ws=0.

  DO 40 i=1,452940
  IF (wn(i).GE.wnaive(j)-del) THEN
  IF (wn(i).LE.wnaive(j)+del) THEN
    wnt=wn(i)
    wnte=wnaive(j)
    delt=del
    window=win(wnt,wnte,delt)
    Ts=Ts + window*spec(i) ! Temporary spectrum value.
    ws=ws + window
  ELSE
    GOTO 35 ! Have got all the bin filled with spectra.
  END IF
  END IF
40  CONTINUE

35  Ts = Ts / ws
30  specave(j)=Ts
  END

```

```

* Triangular window function to use for spectral smoothing.
* wn is the desired wavenumber for the window.
* wnc is the center wavenumber of the bin.
* del is the FWHM of the spectral bin.
* All inputs should be in the same units.
  REAL FUNCTION win(wn,wnc,del)
  IMPLICIT REAL (a-h,o-z)

  IF (wn.EQ.wnc) THEN
    win=1.
  ELSE IF ((wn.GE.wnc-del).AND.(wn.LT.wnc)) THEN
    win=1.-(wnc-wn)/del
  ELSE IF ((wn.GT.wnc).AND.(wn.LE.wnc+del)) THEN
    win=1.-(wn-wnc)/del
  ELSE
    win=0.
  END IF
  RETURN
  END

```

4. Generalization to slant paths through the atmosphere.

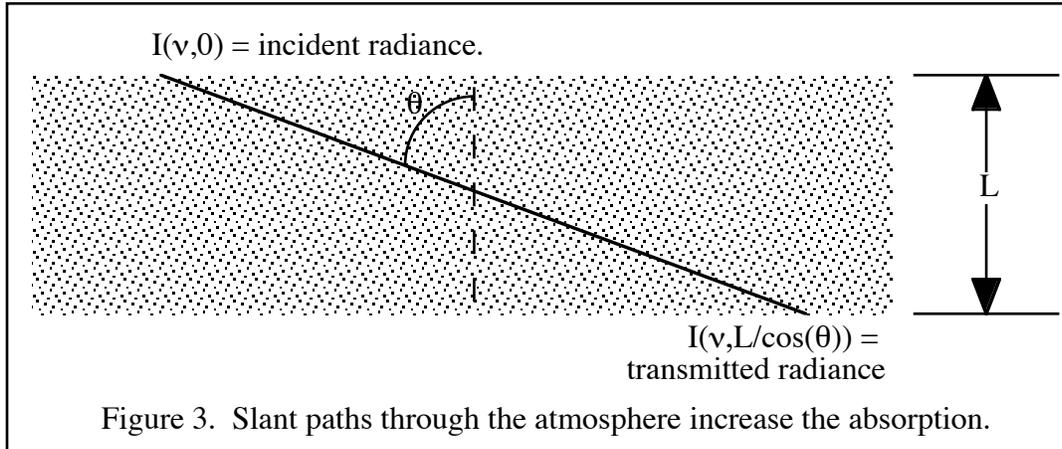


Figure 3 shows a slant path through the atmosphere. Absorption is larger for slant paths simply because the path is longer. For a slant path through the atmosphere, we may compute the transmission from

$$T(\nu, L, \theta) \equiv \frac{I(\nu, 0)}{I(\nu, L / \cos(\theta))} = e^{-\tau_{\text{abs}}(\nu, L / \cos(\theta))} = e^{-\alpha(\nu) L / \cos(\theta)} \quad (11)$$

$$\equiv e^{-\alpha(\nu) L / \mu}$$

where we have introduced the common notation $\mu = \cos(\theta)$. The downward radiance for a slant path can be computed from Eqs. (6) and (7) as before, though with the modification indicated in Eq. (11). Simple.

5. Outgoing radiance at the top of the atmosphere (TOA).

The contribution at the TOA (where the a satellite equipped with a down-looking FTIR is located) of the upward radiance emitted by the m th atmospheric layer, and partially absorbed by upper layers, is

$$I^{(m)}(\nu, \mu) = B(T^{(m)}) \left[\frac{mW}{m^2 \text{ Srcm}^{-1}} \right] \left(1 - e^{-\tau_{\text{abs}}^{(m)}(\nu, L^{(m)} / \mu)} \right) e^{-\sum_{i=m+1}^M \tau_{\text{abs}}^{(i)}(\nu, L^{(i)} / \mu)} \quad (12)$$

The total upward radiance is

$$\begin{aligned}
 I(\nu, \mu) &= \left\{ \sum_{m=0}^M I^{(m)}(\nu, \mu) \left[\frac{\text{mW}}{\text{m}^2 \text{Sr cm}^{-1}} \right] \right\} + \epsilon^{(\text{surface})}(\nu) B(T^{(\text{surface})}) \left[\frac{\text{mW}}{\text{m}^2 \text{Sr cm}^{-1}} \right] e^{-\sum_{i=0}^M \tau_{\text{abs}}^{(i)}(\nu, L^{(i)}) / \mu} \\
 &= \left\{ \sum_{m=0}^M I^{(m)}(\nu, \mu) \left[\frac{\text{mW}}{\text{m}^2 \text{Sr cm}^{-1}} \right] \right\} + \epsilon^{(\text{surface})}(\nu) B(T^{(\text{surface})}) \left[\frac{\text{mW}}{\text{m}^2 \text{Sr cm}^{-1}} \right] TT(\nu, \mu)
 \end{aligned} \quad , \quad (13)$$

where the second form of the equation is used to define the total transmission through the atmosphere, $TT(\nu, \mu)$, $\epsilon^{(\text{surface})}(\nu)$ is the emissivity of the earth's surface (depending on ocean or land, snow, bare, soil type, etc), and the second term is broadly the radiance emitted by the earth that is successfully transmitted through the atmosphere.

6. Irradiance or flux.

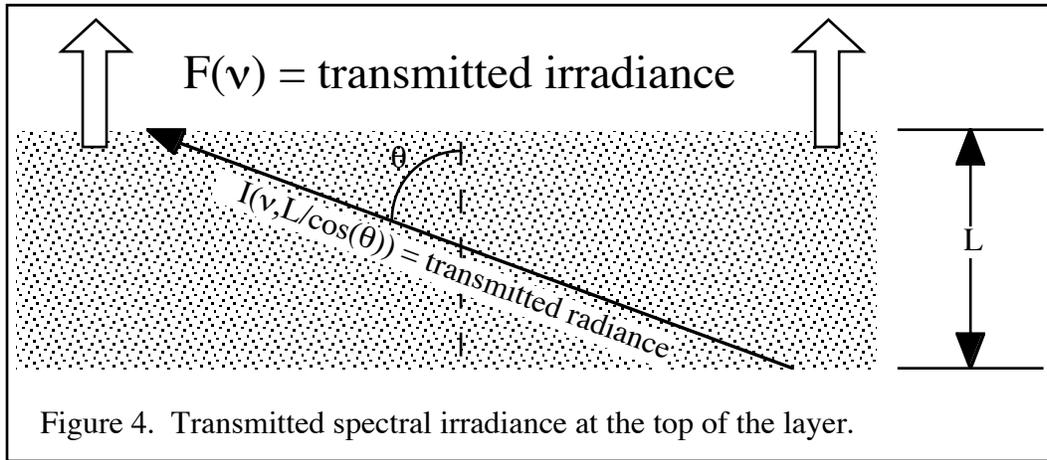


Figure 4. Transmitted spectral irradiance at the top of the layer.

The transmitted irradiance or flux at the top of a layer is indicated schematically in Fig. 4. The relationship between irradiance and radiance for the flat geometry is

$$\begin{aligned}
 F(\nu) &= 2\pi \int_0^{\pi/2} I(\nu, \cos(\theta)) \cos(\theta) \sin(\theta) d\theta \\
 &= 2\pi \int_0^1 I(\nu, \mu) \mu d\mu
 \end{aligned} \quad (14)$$

The total flux at the TOA, for example, is obtained by integration over all wavenumbers,

$$F = \int_0^{\infty} F(\nu) d\nu . \quad (15)$$

We may assign an equivalent radiative temperature to this flux through use of the Stefan Boltzmann relationship,

$$T_{\text{eq}} = \left(\frac{F}{\sigma} \right)^{\frac{1}{4}} . \quad (16)$$

The total flux is important because it is the component of radiance that is in the vertical direction. Changes in the total flux (divergence of the net flux) as a function of altitude give rise to layer heating. The flux at the TOA is important for climatic radiative forcing issues. Flux does not diverge in the horizontal direction because the model here does not include it; we are considering a 1-D radiative problem; however, in the real atmosphere, horizontal flux divergence is caused by horizontal inhomogeneity and curvature of the earth.

7. Climate impact.

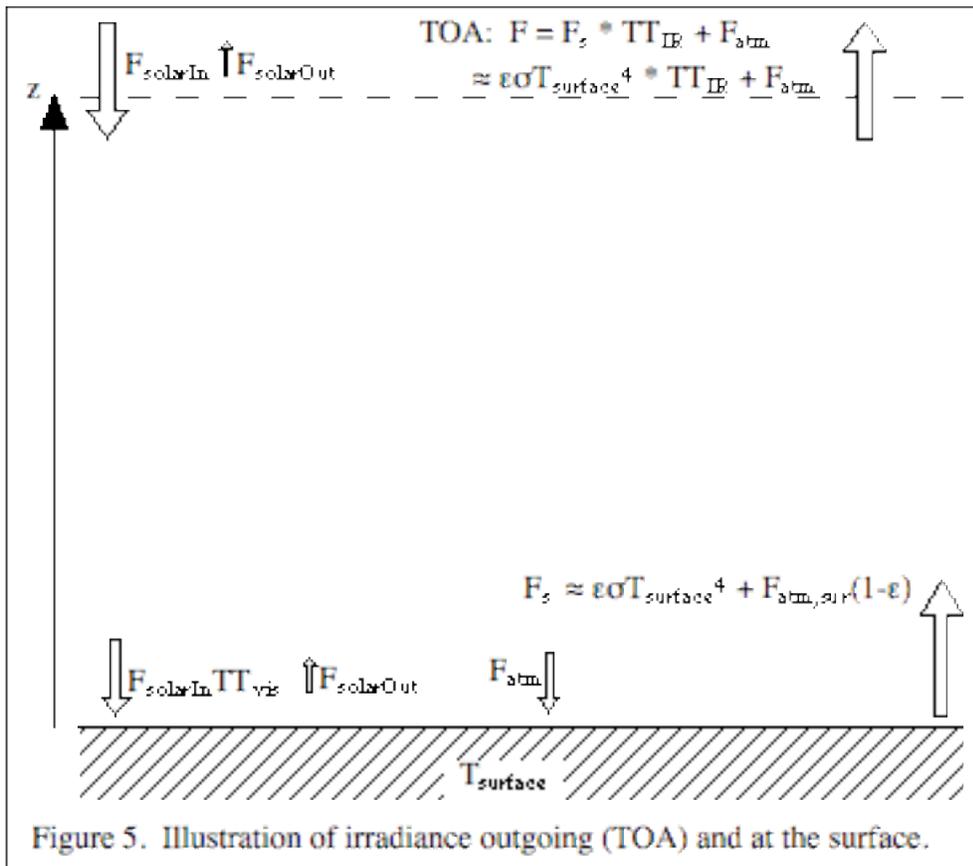


Figure 5 shows some of the relevant fluxes, and includes the solar fluxes for discussion purposes.

The whole point of 'greenhouse gases' is that for a net amount of solar radiation being absorbed by the earth, an energy balance must be achieved, and that 'greenhouse gases' like CO_2 and H_2O reduce the amount of outgoing IR. This balance can be evaluated at the TOA where a known solar flux is coming in and some is reflected, as determined by the global albedo, A , and this is balanced by an amount of IR flux going to space. The solar flux is considered incident on the projected area of the earth, while the outgoing IR flux exits through the entire surface area. One scenario is that a radiative balance could be achieved by increasing the surface temperature to restore the amount of IR radiation in an elevated CO_2 atmosphere. For instance, the approximate earth blackbody radiance necessary to balance the incoming solar flux for $\epsilon \approx 1$ is

$$T_{\text{surface}} \approx \left(\frac{F_{\text{SolarIn}}(1 - A) - 4F_{\text{atm}}}{4\sigma TT_{\text{IR}}} \right)^{\frac{1}{4}} . \quad (17)$$

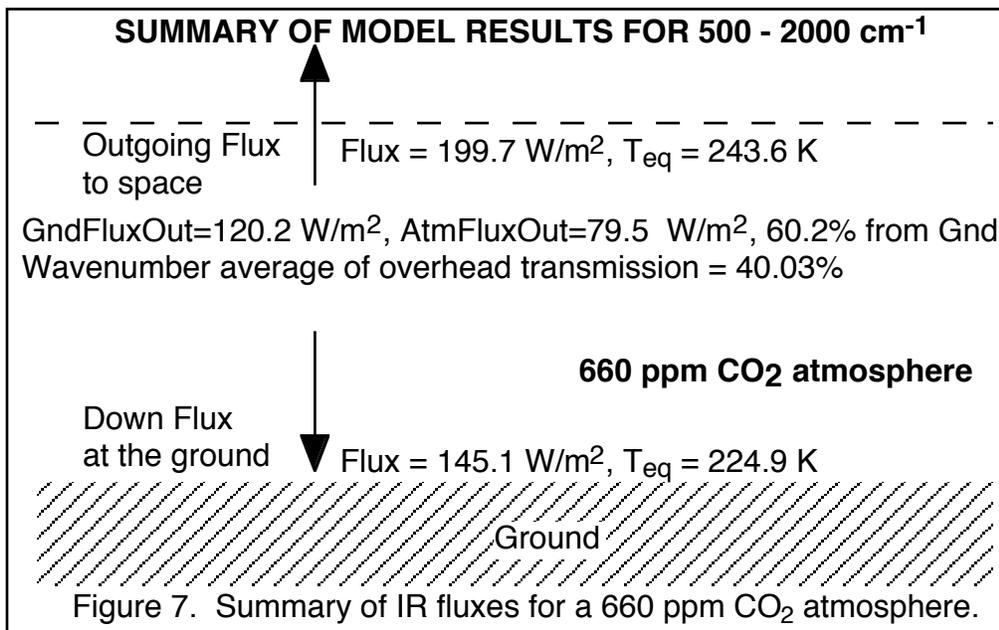
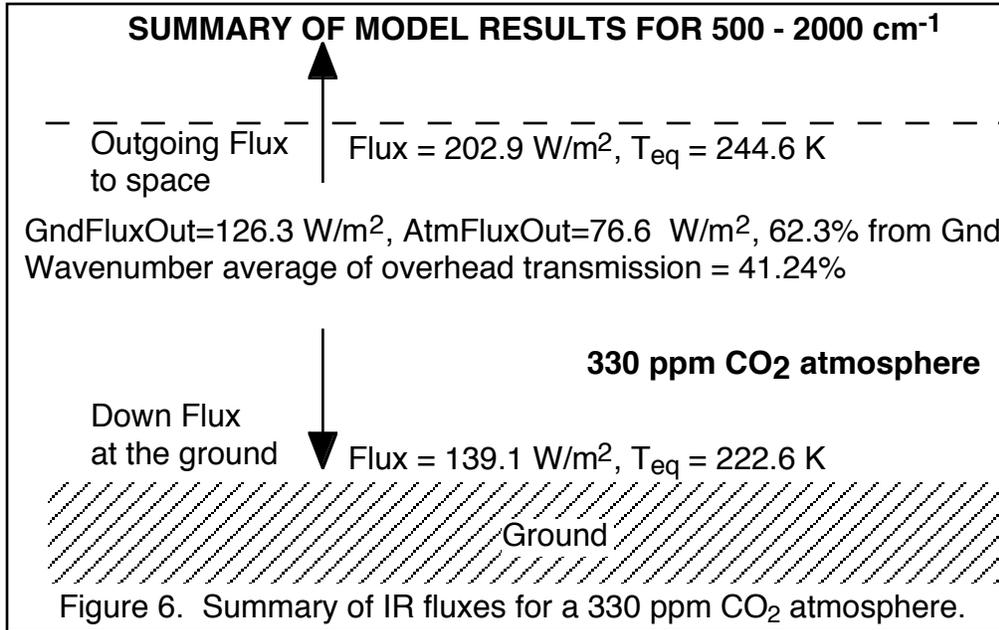
As the atmospheric transmission TT_{IR} diminishes, the temperature at the surface could rise to balance it. Of course the flux emitted by the atmosphere, F_{atm} also will change, so the balance is at once seen to be complex.

Figures 6 and 7 show the summary of IR fluxes for 330 and 660 ppm CO_2 atmospheres, respectively. Comparing the 330 ppm and 660 ppm atmospheres, note that the 660 ppm atmosphere results in less out going flux, and a lower equivalent earth temperature at the TOA, but more down going flux and a warmer equivalent surface temperature.

DANGER:

Water vapor is a very strong ‘greenhouse’ gas. CO_2 may very well change the distribution and amount of water vapor, so is both a greenhouse gas, and a lever arm that affects water vapor content. Recall that saturation vapor pressure of water vapor is an exponential function of temperature -- a little temperature increase can be important for water vapor.

Looking at the effects of CO_2 change in isolation of the bigger picture is dangerous.



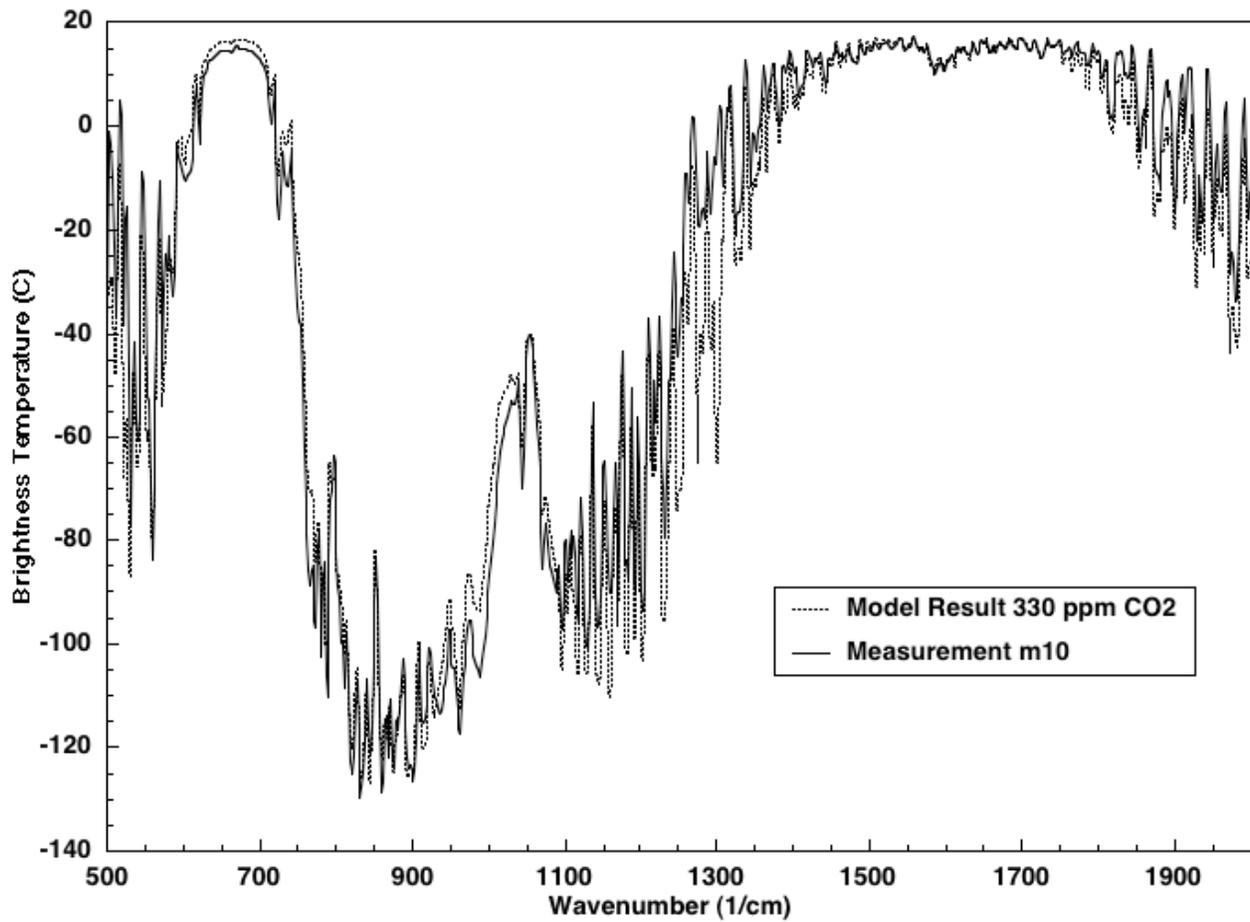


Figure 8. Measurement M10 and approximate model results for the down going overhead radiance. Methane was not included in the model, as evidenced by the error at 1300 cm^{-1} .

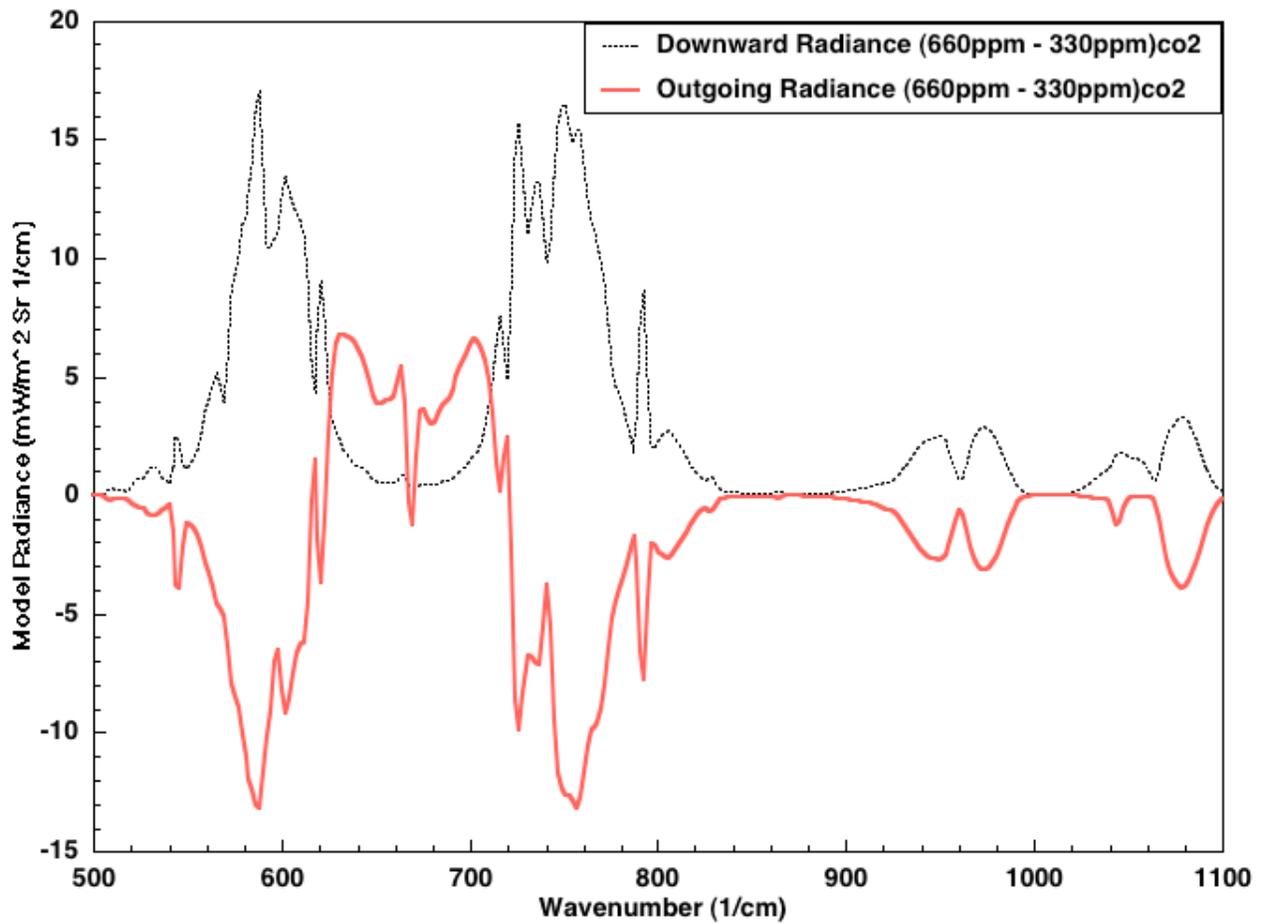


Figure 9. Calculation of the overhead radiance differences at the surface and TOA for CO₂ concentrations of 330 and 660 ppm.